CHAPTER 3

FORCE AND NEWTON'S LAWS

In Chapter 2 we studied the motion of a particle. We did not ask what "caused" the motion; we simply described it in terms of the particle's position, velocity, and acceleration. In this chapter, we discuss the causes of motion, a field of study called dynamics.

The approach to dynamics we consider, which is generally known as classical mechanics, *was developed and successfully tested in the 17th and 18th centuries. More recent theories (special and general relativity and quantum mechanics) have indicated certain realms far from our ordinary experiences where classical mechanics fails to give predictions that agree with experiment, but these new theories reduce to classical mechanics in the limits of ordinary objects.*

Without reference to special or general relativity or to quantum mechanics, we can build great skyscrapers and study the properties of their construction materials; build airplanes that can carry hundreds of people and fly halfway around the world; and send space probes on complex missions to the comets, the planets, and beyond. This is the stuff of classical mechanics.

3-1 CLASSICAL MECHANICS

Ancient philosophers were perplexed by the motion of objects. They wrestled with questions such as: Do all motions require a cause? If so, what is the nature of this cause? Confusion about these issues persisted until the 17th century, when Galileo (1564–1642) and Isaac Newton (1642–1727) developed the approach to understanding these motions that we call "classical mechanics." Newton presented his three laws of motion in 1687 in his *Philosophiae Naturalis Principia Mathematica,* usually called the *Principia.* Until the 20th-century discoveries of quantum physics (which governs the behavior of microscopic particles such as electrons and atoms) and special relativity (which governs the behavior of objects moving at high speed) revealed their limitations, Newton's laws of classical mechanics formed the basis of our understanding of motion and its causes.

In classicial mechanics, we focus our attention on the motion of a particular object, which interacts with the surrounding objects (its *environment*) so that its velocity changes—an acceleration is produced. Table 3-1 shows some common accelerated motions and the object in the environment that is mostly responsible for the acceleration. The central problem of classical mechanics is: (1) An object with known physical properties (mass, volume, electric charge, etc.) is placed at a known initial location moving with known initial velocity. (2) We know (or can measure) all of the interactions of this object with its environment. (3) Can we predict the subsequent motion of the object? That is, can we find its position and velocity at all future times?

For this analysis we will begin by treating physical objects as *particles,* by which we mean bodies whose internal structures or motions can be ignored and whose parts all move in exactly the same way. Often we must analyze the motion of extended objects whose different parts may have different interactions with the environment. For example, a worker might be pushing on one side of a heavy crate while the bottom experiences friction as it slides along the floor. If

TABLE 3-1 Some Accelerated Motions and Their Major Causes

Object	*Change in Motion*	*Object in Environment*	*Type of Force*
Apple	Falls from tree	Earth	Gravitational
Car	Comes to a stop	Road	Frictional
Compass needle	Rotates toward north	Earth	Magnetic
Beam of ink drops in printer	Deflects	Capacitor	Electric
Helium balloon	Rises from land	Air	Buoyant

all parts of the crate move in the same way, we can treat the crate as a particle. As a result, it doesn't matter where the environment acts on the object; our main concern is with the *net effect* of all of the interactions with the environment. (Later in the text we will encounter situations in which it *does* matter where we apply the forces to an extended object, but for now we'll treat all objects as particles.)

We describe the interaction of a body with its environment in terms of a *force* $\vec{\mathbf{F}}$. A force is a push or a pull in a particular direction. Forces are described using vectors—for every force we must specify the direction in which it acts, and forces must be combined using the rules for vector addition. In this chapter we consider mostly situations involving one-dimensional motion, in which case we must specify the force component (positive or negative) relative to that one direction.

Each force that is exerted on an object is caused by a particular body in its environment. As you begin your study of classical mechanics, you may find it useful whenever you are analyzing the forces in a problem to describe each force by the body on which it acts and the body in the environment that is responsible for the force. For example, "pushing force *on* crate *by* worker" or "frictional force *on* crate *by* floor" or "gravitational force *on* crate *by* Earth." You will find this technique especially helpful when we discuss Newton's third law later in this chapter.

To carry out our scheme for classical mechanics, we begin by defining the magnitude of a force in terms of the acceleration of a particular standard body upon which that force acts. We then assign a *mass m* to a body by comparing the acceleration of that body with the acceleration of the standard body when the same force is applied to both. Finally, we develop *force laws* based on the properties of the body and its environment. Force thus appears in both the laws of motion (which tell us what acceleration an object will experience under the action of a given force) and in the force laws (which tell us how to calculate the force on a body in a certain environment). The laws of motion and the force laws together make up the laws of classical mechanics, as Fig. 3-1 suggests.

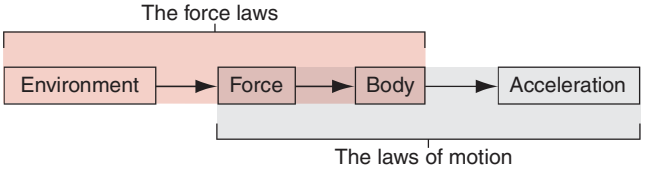

FIGURE 3-1. Our program for mechanics. The three boxes on the left suggest that force is an interaction between a body and its environment. The three boxes on the right suggest that a force acting on a body will accelerate it.

This program of mechanics cannot be tested piecemeal. We must view it as a whole and judge its success based on the answers to two questions: (1) Does the program yield results that agree with experiment? (2) Are the force laws simple and reasonable in form? It is the crowning glory of classical mechanics that we can answer an enthusiastic "yes" to both of these questions.

3-2 NEWTON'S FIRST LAW

Before Galileo's time most philosophers thought that some influence or "force" was needed to keep a body moving. They thought that a body was in its "natural state" when it was at rest. For a body to move in a straight line at constant speed, for example, they believed that some external agent had to continually propel it; otherwise it would "naturally" stop moving.

If we wanted to test these ideas experimentally, we would first have to find a way to free a body from all influences of its environment or from all forces. This is hard to do, but in certain cases we can make the forces very small. If we study the motion as we make the forces smaller and smaller, we can get an idea of what the motion would be like if the external forces were truly zero.

Let us place our test body—say, a block—on a rigid horizontal plane. If we let the block slide along this plane, we note that it gradually slows down and stops. This observation was used, in fact, to support the idea that motion stopped when the external force, in this case the hand initially pushing the block, was removed. We can argue against this idea, however, by reasoning as follows. Let us repeat our experiment, now using a smoother block and a smoother plane and providing a lubricant. We note that the velocity decreases more slowly than before. Let us use still smoother blocks and surfaces and better lubricants. We find that the block decreases in velocity at a slower and slower rate and travels farther each time before coming to rest. You may have experimented with an air track, on which objects can be made to float on a film of air; such a device comes close to the limit of no friction, as even a slight tap on one of the gliders can send it moving along the track at a slow and almost constant speed. We can now extrapolate and say that if all friction could be eliminated, the body would continue indefinitely in a straight line with constant speed. An external

force is needed to set the body in motion, but *no external force is needed to keep a body moving with constant velocity.*

It is difficult to find a situation in which no external force acts on a body. The force of gravity acts on an object on or near the Earth, and resistive forces such as friction or air resistance oppose motion on the ground or in the air. Fortunately, we need not go to the vacuum of distant space to study motion free of external force, because, as far as the overall translational motion of a body is concerned, *there is no distinction between a body on which no external force acts and a body on which the sum or resultant of all the external forces is zero.* We usually refer to the resultant of all the forces acting on a body as the "net" force. For example, the push of our hand on the sliding block can exert a force that counteracts the force of friction on the block, and an upward force of the horizontal plane counteracts the force of gravity. The net force on the block can then be zero, and the block can move with constant velocity.

Note that, even though four forces act on the block, the *net force* can still be zero. The net force is determined by the *vector* sum of all the forces that act on the object. Forces of equal magnitude and opposite direction have a vector sum of zero. Thus we can achieve a condition of no net force on an object by arranging to apply forces that counteract other forces that act on the body, such as a push by a hand or an engine to overcome friction.

This principle was adopted by Newton as the first of his three laws of motion:

> *Consider a body on which no net force acts. If the body is at rest, it will remain at rest. If the body is moving with constant velocity, it will continue to do so.*

The First Law and Reference Frames

Suppose you are a passenger riding in a car and you are tightly held in your seat by the seat belt. When the brakes are applied, a book that was on the seat next to you begins to slide forward. There is no apparent force on the book that is pushing it forward, but relative to you it appears to start moving, in violation of Newton's first law. Your friend Bill, who is standing along the side of the road, sees you, the car, and the book all moving together, say at 22 m/s (about 50 mi/h). If you and the car suddenly slow to 20 m/s (about 45 mi/h), in the absence of friction with the seat the book continues to move at 22 mph according to Bill. Bill notices nothing unusual and detects no violation of Newton's first law.

For another example, you are tightly held by your seat belt in an airplane that encounters turbulence and suddenly drops in altitude by one meter. The glass on your tray table appears to you to leap one meter into the air, with no apparent force causing its motion. From your perspective, it appears that Newton's first law has been violated. Your friend Sally is flying at constant velocity in a plane just next to yours; her plane is not affected by the turbulence. Sally sees your glass moving in a straight line, while you and the plane suddenly drop by one meter. Sally detects no violation of Newton's first law in the motion of your glass.

Each observer—such as you in the car or airplane, Bill standing on the ground, and Sally in her airplane—defines a *reference frame.* A reference frame requires a coordinate system and a set of clocks, which enable an observer to measure positions, velocities, and accelerations in his or her particular reference frame. Observers in different reference frames may measure different velocities or accelerations.

Newton's first law, which may seem like an obvious result, is very important because it helps us to identify a set of special reference frames in which we can apply the laws of classical mechanics. In the example involving the car, you and Bill will reach different conclusions about the acceleration of the book—you conclude that it accelerates forward, while Bill concludes that its acceleration is zero. In general, the acceleration of a body depends on the reference frame relative to which it is measured. However, the laws of classical mechanics are valid only in a certain set of reference frames—namely, those in which *all* observers would measure the *same* acceleration for a moving body. Newton's first law allows us to choose this special family of reference frames if we express it as follows:

> *If the net force acting on a body is zero, then it is possible to find a set of reference frames in which that body has no acceleration.*

The tendency of a body to remain at rest or in uniform linear motion is called *inertia,* and Newton's first law is often called the *law of inertia.* The reference frames to which it applies are called *inertial frames.*

To test whether a particular frame of reference is an inertial frame, we place a test body at rest in the frame and ascertain that no net force acts on it. If the body does not remain at rest, the frame is not an inertial frame. Similarly, we can put the body (again subject to no net force) in motion at constant velocity; if its velocity changes, either in magnitude or direction, the frame is not an inertial frame. A frame in which these tests are everywhere passed is an inertial frame.

As a passenger in the decelerating car, your frame of reference is not an inertial frame, and you cannot directly apply the laws of mechanics as we formulate them. If Bill's frame of reference passes the tests as an inertial frame, he can successfully apply the laws of mechanics in his frame. He can measure the change in velocity of the car and thus deduce its acceleration (due to the force of friction with the road), but he concludes that the net force on the book is zero and thus that it should move with constant velocity. Similarly, if Sally's frame of reference passes the tests, she can successfully apply the laws of mechanics by associating your plane's sudden change in vertical velocity with a net vertical force (in this case, the difference between gravity and the upward "lift" force), and she can account for the motion of the glass by applying those same laws.

Inertial Reference Frames and Relative Motion

Suppose you are in a car moving down the road at 22 m/s (about 50 mi/h). Your friends in the same car are also moving at 22 m/s. You could toss a ball sideways into your friend's lap, and the toss is unaffected by the velocity of the car. As long as the car continues to move at constant velocity, the ball lands in your friend's lap.

If you are passed by your friend Sally who is in another car moving with a velocity of 27 m/s (about 60 mi/h), you observe the distance between your car and Sally's to increase at the rate of 27 m/s − 22 m/s = 5 m/s. That is, relative to your car, Sally's car is moving at 5 m/s. Take away the external clues—the scenery speeding by, the still air rushing past the moving car, the bumpiness of the road, and the noise of the engine—and consider only the two cars. You would have no way to decide which car was "really" moving. For example, Sally's car could be at rest and your car could be moving backward at 5 m/s; the observed result would be the same. One minute after she passes you, you observe the distance between you and Sally to be the relative velocity times the time interval: 5 m/s × 60 s = 300 m.

Now consider how this looks to your friend Bill by the side of the road (Fig. 3-2). Suppose Sally passes you just as you both pass Bill's position. According to Bill, 1 minute later your car has moved a distance of 22 m/s × 60 s = 1320 m, while Sally's car has moved a distance of 27 m/s × 60 s = 1620 m. Bill concludes that the distance between the cars is 1620 m − 1320 m = 300 m. Thus you and Bill agree on your conclusions about the distance between the cars.

Soon after passing you, Sally sees a police car and applies her brakes. Bill observes her to slow from 27 m/s to 20 m/s in a time of 3.5 s. According to Bill, her acceleration is (taking the direction of her motion to be the positive x direction and solving Eq. 2-26, $v_x = v_{0x} + a_x t$, for a_x)

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{(20\text{ m/s}) - (27\text{ m/s})}{3.5\text{ s}} = -2.0\text{ m/s}^2.$$

According to your frame of reference, Sally's velocity is now 20 m/s − 22 m/s = −2 m/s; that is, you are now moving faster than Sally by 2 m/s. According to you, her velocity changes from +5 m/s to −2 m/s and thus her acceleration is

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{(-2\text{ m/s}) - (5\text{ m/s})}{3.5\text{ s}} = -2.0\text{ m/s}^2.$$

You and Bill agree on the acceleration!

You and Bill are both inertial observers. You agree on the acceleration of Sally's car, and you will therefore agree on the force that was necessary to cause the acceleration. In fact, *all inertial observers agree on measurements of acceleration* (although they will not in general agree on measurements of position or velocity).

Consider the contrary case in which you also brake slightly when passing the police car, reducing your speed from 22 m/s to 21 m/s in the same 3.5-s interval. At the beginning of the interval, you determine Sally's velocity to be +5 m/s, as before. At the end of the braking interval, you would determine her velocity to be 20 m/s − 21 m/s = −1 m/s (in the backward direction). You would then conclude Sally's acceleration to be

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{(-1\text{ m/s}) - (5\text{ m/s})}{3.5\text{ s}} = -1.7\text{ m/s}^2,$$

which differs from Bill's result of −2.0 m/s^2. You are no longer an inertial observer (because during the 3.5-s interval in which you were braking you could no longer pass the test of Newton's first law).

In this book we almost always apply the laws of classical mechanics from the point of view of an observer in an inertial frame. Occasionally, we discuss problems involving observers in noninertial reference frames, such as an accelerating car, a rotating merry-go-round, or an orbiting satellite. Even though the Earth is rotating, a reference frame attached to the Earth can be considered to be approximately an inertial reference frame for most practical purposes. For large-scale applications, such as analyzing the flight of ballistic missiles or studying wind and ocean currents, the noninertial character of the rotating Earth becomes important. (See Section 5-6.)

Note that there is no distinction in the first law between a body at rest and one moving with a constant velocity. Both motions are "natural" if the net force acting on the body is zero. This becomes clear when a body at rest in one inertial frame is viewed from a second inertial frame—that is, a frame moving with constant velocity with respect to the first. An observer in the first frame finds the body to be at rest; an observer in the second frame finds the same body to be moving with constant velocity. Both observers find the body to have no acceleration—that is, no change in velocity—and both may conclude from the first law that no net force acts on the body.

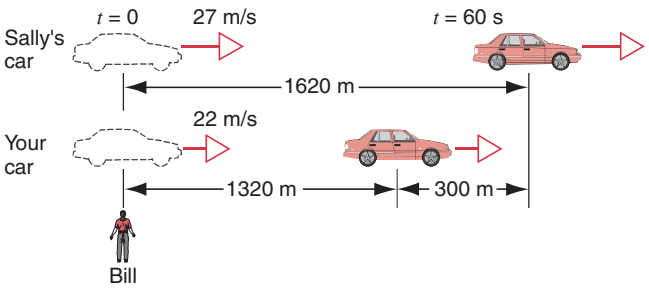

FIGURE 3-2. You and Bill agree that Sally's car is 300 m ahead of yours after 60 s.

3-3 FORCE

According to Newtons' first law, the *absence* of force leads to the *absence* of acceleration. What about the *presence* of force? Based on common experience, it is reasonable to as-

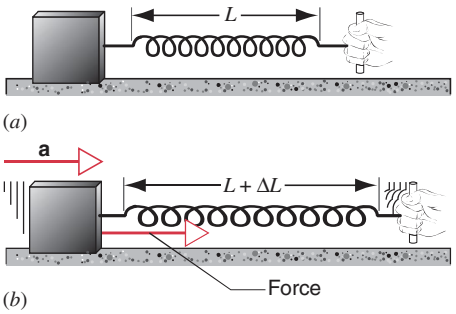

FIGURE 3-3. (*a*) A standard body at rest on a horizontal frictionless surface. (*b*) The body is accelerated by pulling to the right to stretch the spring by ΔL.

sume that a body will accelerate when a force is applied to it. We now develop our concept of force by defining it operationally in terms of the acceleration it produces when applied to a chosen standard body. Any object can serve as our "standard body," as long as it is clearly identifiable and reproducible. For example, we might choose a block of copper or glass of any specified dimensions.

Before we proceed with this measurement, we must first invoke Newton's first law to check that we are working in an inertial reference frame. If the body is at rest, does it remain at rest? If we start it moving with a constant velocity, does it remain in that state of motion? In attempting to answer the second question, we probably will discover that the body, once in motion, gradually slows down due to friction. By careful design of our apparatus, we create as nearly a friction-free environment as we can, perhaps by floating the body on a film of air or lubricating the surface on which it moves. We intend to apply a force to the body and measure its acceleration, and we want to be sure that any other forces caused by the environment have a negligible effect on the motion of the body.

As the agent that supplies the force, we choose a light spring. We observe that springs come in different stiffnesses, requiring different efforts to stretch them. We also observe that the effort needed to stretch a given spring increases as we stretch it through larger distances.

We place the standard body on a frictionless horizontal surface and attach the spring to it (Fig. 3-3*a*). By trial and error we stretch our spring until it gives an acceleration of exactly 1 m/s^2 to our standard body (Fig. 3-3*b*). We define this as *one unit* of force, and for future reference we record the amount of stretch ΔL in the spring that corresponds to this force. Now we repeat the experiment, stretching the spring by a greater amount until the standard body experiences an acceleration of 2 m/s^2. We define this as *two units* of force, and we again record the amount of stretch ΔL for the spring that gives this force. Continuing, we find the amount of stretch that gives an acceleration of 3 m/s^2, corresponding to *three units* of force. Eventually we have a complete calibration of our spring, giving the amount of stretch that gives our standard body any desired acceleration. Other springs of different stiffnesses could be calibrated in a similar way using the same standard body.

Based entirely on the acceleration given to the standard body, we now have a calibrated set of springs. Using these measuring devices, we can now proceed to measure unknown forces. For example, let us suspend an object vertically from a spring, as in Fig. 3-4*a*. Since the body is at rest, the net force on it must be zero. The magnitude of the upward force exerted by the spring must equal the magnitude of the downward force of gravity, so that the vector sum of the two forces is zero. By measuring the extension ΔL and checking our calibration for that spring to determine the corresponding force, we can determine the force of gravity on the body. In fact, calibrated spring scales are available for just this purpose, such as to weigh fruits and vegetables in grocery stores.

In a similar way we can measure frictional forces. By placing the body on a horizontal surface where it experiences a frictional force, we could attach a spring (as in Fig. 3-4*b*) and pull the body with just the right force so that it moves at constant velocity. In this case, the magnitude of the spring force equals that of the frictional force, so their vector sum is zero (because they have equal magnitudes and opposite directions). We can determine the magnitude of the force from the amount by which the spring is stretched. Once again, a calibrated spring scale can be used for this purpose.

Another way to measure force is to use a (commercially available) electronic *force probe,* which can be interfaced to a computer to read forces directly (see Fig. 3-5). A force applied to the probe causes a small deflection of a mechani-

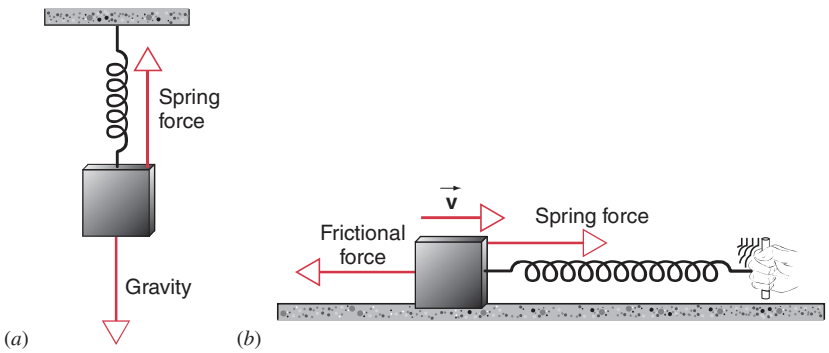

FIGURE 3-4. (*a*) A body is suspended at rest, acted upon by the spring force and gravity. (*b*) A body moves at constant velocity on a horizontal surface where a frictional force is exerted upon it.

FIGURE 3-5. A mass hangs from a spring attached to an electronic probe, which can be interfaced to a computer for measuring forces. Courtesy Vernier Software and Technology.

cal or electromagnetic device; the deflection can be read electronically and calibrated against a "standard" spring.

3-4 MASS

In the previous section we discussed a series of experiments that we used to calibrate a set of springs based on the accelerations given to a standard body when the springs were stretched to different lengths. Now we want to repeat those experiments to answer a different question: What effect will the *same force* have when applied to *different bodies?*

Everyday experience leads us to guess at a qualitative answer: it is much easier to accelerate a bicycle than a car by pushing it. Clearly the same force produces different accelerations when applied to different bodies. What makes these bodies differ in our ability to accelerate them by pushing is their *mass,* which is *the property of a body that determines its resistance to a change in its motion.*

Let us see how we can study the relationship between force and mass by accelerating bodies of different masses using our calibrated spring set. We start by obtaining a second identical standard body and attaching it to the first body. We then apply one unit of force (as previously determined from the single body) to this combined object, and we observe the acceleration to be 0.5 m/s^2. Applying two units of force, we find an acceleration of 1.0 m/s^2. We can repeat the experiment with three standard bodies joined together, then four, and so forth. Here is a possible set of outcomes of these experiments:

Applied Force	1 unit	2 units	3 units	4 units
Acceleration of one standard body	1.0 m/s^2	2.0 m/s^2	3.0 m/s^2	4.0 m/s^2
Acceleration of two standard bodies	0.5 m/s^2	1.0 m/s^2	1.5 m/s^2	2.0 m/s^2
Acceleration of three standard bodies	0.33 m/s^2	0.67 m/s^2	1.0 m/s^2	1.3 m/s^2

It is clear from the results of these measurements that, for each combination of bodies, the acceleration is directly proportional to the force (for example, in each case two units of force gives twice the acceleration as one unit of force). The proportionality constant between force and acceleration, however, is different for each horizontal row of the data table and is a characteristic property of the object being accelerated. This property of the object is its mass, which gives the proportionality between force and acceleration. Figure 3-6 illustrates these experiments, which show the relationships among force, mass, and acceleration.

From many experiments similar to these, we conclude that the greater the total mass of a body, the smaller the acceleration produced by a given force. That is, *the acceleration produced by a given force is inversely proportional to the mass being accelerated.* The mass of a body can thus be regarded as *a quantitative measure of the resistance of a body to acceleration by a given force.*

This observation gives us a direct way to compare the masses of different bodies: we apply the same force to both bodies and measure the resulting acceleration. The ratio of the masses of the two bodies is then the same as the *inverse* ratio of the accelerations. For example, suppose we apply a force F to the standard body (whose mass we take to be m_{std}) and measure acceleration a_{std}. We then observe that the same force F applied to body x of unknown mass m_x gives acceleration a_x. Forming the ratios, we then have

$$\frac{m_x}{m_{\text{std}}} = \frac{a_{\text{std}}}{a_x} \qquad \text{(same force } F \text{ acting).} \qquad (3\text{-}1)$$

This allows us to find the mass of the unknown body in terms of the mass of our chosen standard body. For example, if the acceleration of body x is $\frac{1}{3}$ of the acceleration of the standard body when the same force is applied to both, then the mass of body x is three times the mass of the standard body. Note that this remains true no matter how many units of force we choose to apply to both bodies, as you can see from the above table of measured values—for example, the accelerations of the triple body in the last line of the table are $\frac{1}{3}$ of the corresponding accelerations of the single

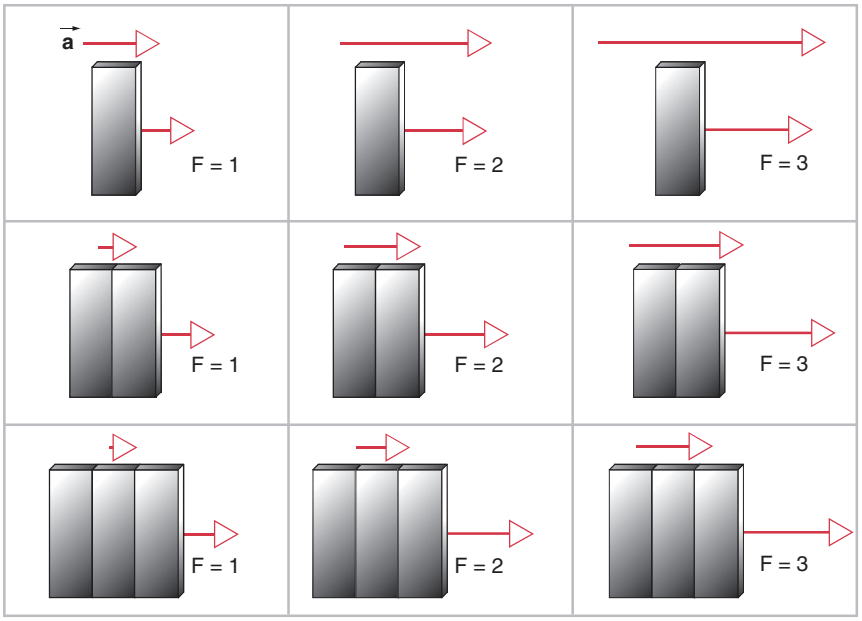

FIGURE 3-6. Experiments illustrating the relationship among force (given in arbitrary units), mass, and acceleration. Acceleration vectors are drawn to scale above the blocks. Reading across each row, we see that the acceleration is always proportional to the force, but the proportionality is different for different masses. Reading down each column, we see that when the same force acts, the acceleration is inversely proportional to the mass.

body for each value of the applied force. More specifically, if we apply a different force F' to both the standard body and body x, giving accelerations a'_{std} and a'_x, we then find that the ratio of the accelerations with force F' is the same as the ratio with force F:

$$\frac{m_x}{m_{\text{std}}} = \frac{a_{\text{std}}}{a_x} = \frac{a'_{\text{std}}}{a'_x}. \qquad (3\text{-}2)$$

We obtain the same value for the unknown mass m_x, *no matter what the value of the common force.* The mass ratio m_x/m_{std} is independent of the force; the mass is a fundamental property of the object, unrelated to the value of the force used to compare the unknown mass to the standard mass.

By a simple extension of this procedure, we can compare the masses of any two bodies with each other, rather than comparing a single body with the standard. Consider two arbitrary objects, of masses m_1 and m_2. We apply a given force of magnitude F to m_1 and measure acceleration a_1. Applying the same force to m_2, we obtain acceleration a_2. The mass ratio is

$$\frac{m_2}{m_1} = \frac{a_1}{a_2} \qquad \text{(same force acting)}, \qquad (3\text{-}3)$$

which turns out to be identical with the ratio we would obtain by deducing the masses m_1 and m_2 separately by direct comparison with the standard, as in Eq. 3-1.

This procedure also shows that, when two masses m_1 and m_2 are fastened together, they behave mechanically like a single object of mass $m_1 + m_2$. This demonstrates that *masses add like (and are) scalar quantities.*

One practical example of the use of this technique—assigning masses by comparing the accelerations produced by a given force—is in the precise measurement of the masses of atoms. The force in this case is magnetic, and the acceleration is perpendicular to the velocity of the atom and so causes a deflection in its path, but the principle is exactly the same: the ratio of the masses of the two atoms is equal to the inverse ratio of their accelerations. Measuring the deflection permits precise mass ratios to be measured, and comparing with a standard mass (that of ^{12}C, defined to be exactly 12 u) permits precise values of masses, such as those shown in Table 1-6, to be obtained.

3-5 NEWTON'S SECOND LAW

We can now summarize all the previously described experiments and definitions in one equation, the fundamental equation of classical mechanics,

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}. \qquad (3\text{-}4)$$

In this equation $\Sigma\,\vec{\mathbf{F}}$ is the (vector) *sum* of *all* the forces acting *on* the body, m is the mass of the body, and $\vec{\mathbf{a}}$ is its (vector) acceleration. We shall usually refer to $\Sigma\,\vec{\mathbf{F}}$ as the *resultant* force or *net* force.

Equation 3-4 is a statement of *Newton's second law.* If we write it in the form $\vec{\mathbf{a}} = (\Sigma\,\vec{\mathbf{F}})/m$, we can easily see that the acceleration of the body is in magnitude directly proportional to the resultant force acting on it and in direction parallel to this force. We also see that the acceleration, for a given force, is inversely proportional to the mass of the body.

Note that the first law of motion appears to be contained in the second law as a special case, for if $\Sigma\,\vec{\mathbf{F}} = 0$, then $\vec{\mathbf{a}} = 0$. In other words, if the resultant force on a body is zero, the acceleration of the body is zero and the body moves with constant velocity, as stated by the first law. However, the first law has an independent and important

role in defining inertial reference frames. Without that definition, we would not be able to choose the frames of reference in which to apply the second law. We therefore need *both laws* for a complete system of mechanics.

Equation 3-4 is a vector equation. As in the case of all vector equations, we can write this single vector equation as three one-dimensional equations,

$$\sum F_x = ma_x, \qquad \sum F_y = ma_y, \qquad \sum F_z = ma_z, \quad (3\text{-}5)$$

relating the x, y, and z components of the resultant force (ΣF_x, ΣF_y, and ΣF_z) to the x, y, and z compoonents of acceleration (a_x, a_y, and a_z) for the mass m. It should be emphasized that ΣF_x is the *algebraic* sum of the x components of *all* the forces, ΣF_y is the *algebraic* sum of the y components of *all* the forces, and ΣF_z is the *algebraic* sum of the z components of *all* the forces acting on m. In taking the algebraic sum, the signs of the components (that is, the relative directions of the forces) must be taken into account.

Like all equations, Newton's second law must be dimensionally consistent. On the right side, the dimensions are, recalling from Chapter 1 that [ ] denotes *the dimensions of,* $[m][a] = \mathrm{ML/T^2}$, and therefore these must also be the dimensions of force:

$$[F] = \mathrm{ML/T^2}.$$

No matter what the origin of the force—gravitational, electrical, nuclear, or whatever—and no matter how complicated the equation describing the force, these dimensions must hold for it.

In the SI system of units, the standard body has a mass of one kilogram (see Section 1-5), and we in effect measure the masses of objects by comparing them with the standard kilogram. To impart an acceleration of 1 m/s^2 to a mass of 1 kg requires a force of 1 kg·m/s^2. This combination of units is called the *newton* (abbreviated N):

$$1\ \mathrm{N} = 1\ \mathrm{kg \cdot m/s^2}.$$

If we measure the mass in kg and the acceleration in m/s^2, Newton's second law gives the force in N.

Two other systems of units in common use are the cgs (centimeter-gram-second) and the British systems. In the cgs system, mass is measured in grams and acceleration in cm/s^2. The force unit in this system is the *dyne* and is equivalent to the g·cm/s^2. Since 1 kg = 10^3 g and 1 m/s^2 = 100 cm/s^2, it follows that 1 N = 10^5 dyne. A dyne is a very small unit, roughly equal to the weight of a cubic millimeter of water. (A newton, on the other hand, is about the weight of a half-cup of water.)

In the British system, force is measured in pounds and acceleration in ft/s^2. In this system, the mass that is accelerated at 1 ft/s^2 by a force of 1 lb is called the *slug* (from the word *sluggish,* meaning slow or unresponsive).

Other variants on these basic systems are occasionally found, but these three are by far the most common. Table 3-2 summarizes these common force units; a more extensive listing can be found in Appendix G.

TABLE 3-2 Units in Newton's Second Law

System	*Force*	*Mass*	*Acceleration*
SI	newton (N)	kilogram (kg)	m/s^2
cgs	dyne	gram (g)	cm/s^2
British	pound (lb)	slug	ft/s^2

Dynamical Analysis Using Newton's Second Law

In analyzing problems using Newton's second law, there are several steps that you should follow:

1. Choose a suitable inertial reference frame. Select the orientation and the positive direction for the coordinate axes in that reference frame. Force components in the positive direction are taken to be positive, and those in the opposite direction are negative.
2. For each object in the problem, draw a *free-body diagram,* showing all of the forces acting on that body. The body is regarded as a particle in this diagram.
3. Label each force with two subscripts: the first one indicates the body on which the force acts, and the second indicates the body in the environment that causes the force. For example, F_{AB} indicates the force on body A due to body B, and F_{BA} indicates the force on body B due to body A. If there are several objects A, B, C . . . in the problem, then the forces on body A might include F_{AB}, F_{AC}, and so forth. This method of labeling the forces is very important, because it will help you avoid making the mistake of including a fictitious force that is not associated with a body in the environment.
4. For each body, find the vector sum of all the forces. In practice, this usually means separately adding (with proper attention to the signs) the x, y, and z components of the forces. Then use Eqs. 3-5 to find the acceleration components of that body.

The following examples illustrate the application of these procedures.

SAMPLE PROBLEM 3-1. A worker W pushes a loaded sled S whose mass m is 240 kg for a distance d of 2.3 m over the surface of a frozen lake. The sled moves with negligible friction on the ice. The worker exerts a constant horizontal force F_{SW} of 130 N (= 29 lb) as she does so; see Fig. 3-7*a*. If the sled starts from rest, what is its final velocity?

Solution As Fig. 3-7*b* shows, we lay out a horizontal x axis, we take the direction of increasing x to be to the right, and we treat

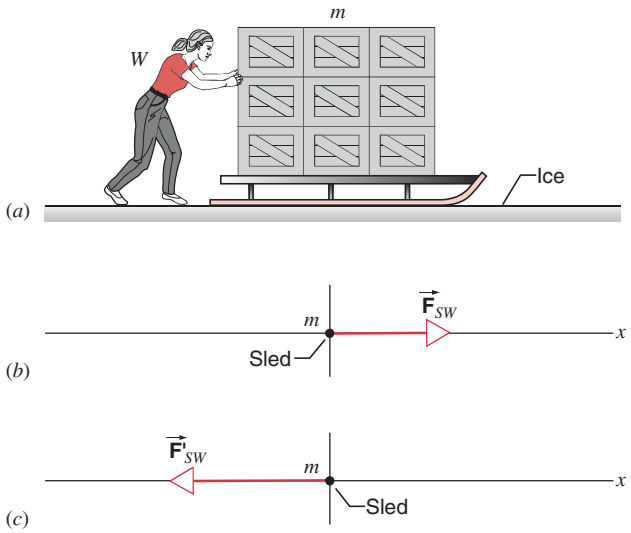

FIGURE 3-7. Sample Problems 3-1 and 3-2. (*a*) A worker pushing a loaded sled over a frictionless surface. (*b*) A free-body diagram, showing the sled as a "particle" and the force acting on it. (*c*) A second free-body diagram, showing the force acting when the worker pushes in the opposite direction.

the sled as a particle. Figure 3-7*b* is a *partial* free-body diagram. In drawing free-body diagrams, it is important always to include *all* forces that act on the particle, but here we have omitted two vertical forces that will be discussed later and that do not affect our solution. We assume that the force F_{SW} exerted by the worker is the only horizontal force acting on the sled, so that $\Sigma F_x = F_{SW}$. We can then find the acceleration of the sled from Newton's second law, or

$$a_x = \frac{\Sigma F_x}{m} = \frac{F_{SW}}{m} = \frac{130\text{ N}}{240\text{ kg}} = 0.54\text{ m/s}^2.$$

With this acceleration, we can find the time necessary to move a distance d using Eq. 2-28 ($x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$) with $x - x_0 = d$ and $v_{0x} = 0$). Solving, we obtain $t = \sqrt{2d/a_x} = 2.9$ s. Equation 2-26 (with $v_{0x} = 0$) now gives the final velocity

$$v_x = a_xt = (0.54\text{ m/s}^2)(2.9\text{ s}) = 1.6\text{ m/s}.$$

The force, acceleration, displacement, and final velocity of the sled are all positive, which means that they all point to the right in Fig. 3-7*b*.

Note that to continue applying the constant force, the worker would have to run faster and faster to keep up with the accelerating sled. Eventually, the velocity of the sled would exceed the fastest speed at which the worker could run, and thereafter she would no longer be able to apply a force to the sled. The sled would then continue (in the absence of friction) to coast at constant velocity.

SAMPLE PROBLEM 3-2. The worker in Sample Problem 3-1 wants to reverse the direction of the velocity of the sled in 4.5 s. With what constant force must she push on the sled to do so?

Solution If she exerts a constant force, then the acceleration of the sled will be constant. Let us find this constant acceleration, using Eq. 2-26 ($v_x = v_{0x} + a_xt$). Solving for a gives

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{(-1.6\text{ m/s}) - (1.6\text{ m/s})}{4.5\text{ s}} = -0.71\text{ m/s}^2.$$

This is larger in magnitude than the acceleration in Sample Problem 3-1 (0.54 m/s^2), so it stands to reason that the worker must push harder this time. We find this (constant) force F'_{SW} from

$$F'_{SW} = ma_x = (240\text{ kg})(-0.71\text{ m/s}^2)$$
$$= -170\text{ N} (= -38\text{ lb}).$$

The negative sign shows that the worker is pushing the sled in the direction of decreasing x—that is, to the left as shown in the free-body diagram of Fig. 3-7*c*.

SAMPLE PROBLEM 3-3. A crate whose mass m is 360 kg rests on the bed of a truck that is moving at a speed v_0 of 105 km/h, as in Fig. 3-8*a*. The driver applies the brakes and slows to a speed v of 62 km/h in 17 s. What force (assumed constant) acts on the crate during this time? Assume that the crate does not slide on the truck bed.

Solution We first find the (constant) acceleration of the crate. Solving Eq. 2-26 ($v_x = v_{0x} + a_xt$) for a_x yields

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{(62\text{ km/h}) - (105\text{ km/h})}{(17\text{ s})(3600\text{ s/h})} = -0.70\text{ m/s}^2.$$

Because we have taken the positive sense of the horizontal direction to the right, the acceleration must point to the left.

The force F_{CT} exerted on the crate by the truck follows from Newton's second law:

$$F_{CT} = ma_x$$
$$= (360\text{ kg})(-0.70\text{ m/s}^2) = -250\text{ N}.$$

This force acts in the same direction as the acceleration—namely, to the left in Fig. 3-8*b*. The force must be supplied by an external agent, such as the straps or other mechanical means used to secure the crate to the truck bed. If the crate is not secured, then friction between the crate and the truck bed must supply the required force. If there is not enough friction to provide a force of 250 N, the crate will slide on the truck bed because, as measured by a ground-based observer, it will slow down less rapidly than the truck.

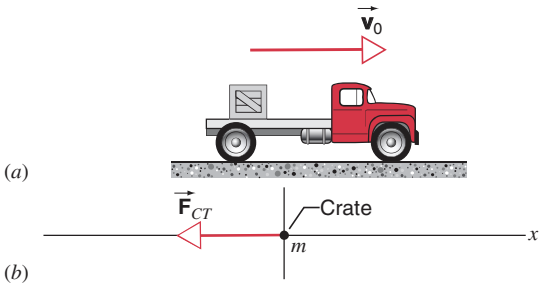

FIGURE 3-8. Sample Problem 3-3. (*a*) A crate on the truck that is slowing down. (*b*) The free-body diagram of the crate.

3-6 NEWTON'S THIRD LAW

Consider the Earth and the Moon. The Earth exerts a gravitational force on the Moon, and the Moon exerts a gravitational force on the Earth. All forces are part of such mutual interactions between two (or more) bodies—it is not possible to have only a single isolated force.

The forces that act on a body (let us call it body A) are due to the other bodies in its environment. Suppose that body B is one of these bodies in the environment of body A. Then among the forces acting on body A is $\vec{\mathbf{F}}_{AB}$, the force on body A due to body B. Alternatively, we might direct our attention to body B. Among the bodies in the environment of body B is body A, which exerts a force $\vec{\mathbf{F}}_{BA}$ on body B. Newton's third law concerns the relationship between $\vec{\mathbf{F}}_{AB}$ and $\vec{\mathbf{F}}_{BA}$.

We find by experiment that when one body exerts a force on a second body, then the second body always exerts a force on the first. Furthermore, we find the forces *always* to be equal in magnitude and opposite in direction. In the Earth–Moon system, the magnitude of the force on the Moon due to the Earth is equal to the magnitude of the force of the Earth due to the Moon. The forces are also opposite in direction—if we imagine a line connecting the Earth and the Moon, then the force on the Moon by the Earth acts along that line toward the Earth and the force on the Earth by the Moon acts along that same line toward the Moon.

Newton's third law summarizes these observations:

> *When one body exerts a force on another, the second exerts a force on the first. These two forces are always equal in magnitude and opposite in direction.*

Formally (see Fig. 3-9), let body B exert a force $\vec{\mathbf{F}}_{AB}$ on body A; experiment then shows that body A exerts a force $\vec{\mathbf{F}}_{BA}$ on body B. These forces are related by

$$\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA}. \tag{3-6}$$

The negative sign reminds us that the forces act in opposite directions, as shown in Fig. 3-9.

It is customary to label the two forces $\vec{\mathbf{F}}_{AB}$ and $\vec{\mathbf{F}}_{BA}$ due to the mutal interaction of two bodies as the "action" and "reaction" forces. These labels are completely arbitrary; either force could be called the "action," and its partner would then be the "reaction." By using these common labels, we do not mean to imply that the "action" somehow causes the "reaction." Both forces exist due to the mutual interaction, and we simply pick one as the "action," which then leaves the other as the "reaction." This gives us a shorthand way of stating Newton's third law:

> *To every action there is an equal and opposite reaction.*

This law requires that the reaction force must exist and it also specifies its magnitude and direction.

It is important to remember that the action and reaction forces *always* act on *different* bodies, as the subscripts remind us. Often you will encounter situations in which two equal and opposite forces act on the same body (as in Fig. 3-4). Equation 3-6 shows that these two forces *cannot* be an action–reaction pair, because they act on the same body. In a true action–reaction pair, one force acts on body A and the other on body B. If you have labeled your forces carefully by specifying the body on which the force acts and the body that causes the force, then you can identify the reaction force by simply interchanging the names of the two bodies. For example:

Action: Force on book due to table	*Reaction:* Force on table due to book
Action: Force on Moon due to Earth	*Reaction:* Force on Earth due to Moon
Action: Force on electron due to nucleus	*Reaction:* Force on nucleus due to electron
Action: Force on baseball by bat	*Reaction:* Force on bat by baseball

If our goal is to study the dynamics of one body (say, the book or the baseball) then only one member of the action–reaction pair would be considered (the force that acts on that body). The other member would be considered only if we were also studying the dynamics of the second body (the table or the bat).

The following examples illustrate applications of the third law.

1. *An orbiting satellite.* Figure 3-10 shows a satellite orbiting the Earth. The only force that acts on it is $\vec{\mathbf{F}}_{SE}$, the

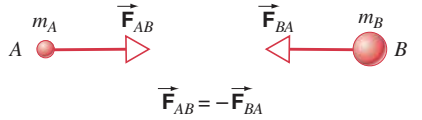

FIGURE 3-9. Newton's third law. Body A exerts a force $\vec{\mathbf{F}}_{BA}$ on body B. Body B must then exert a force $\vec{\mathbf{F}}_{AB}$ on body A, and $\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA}$.

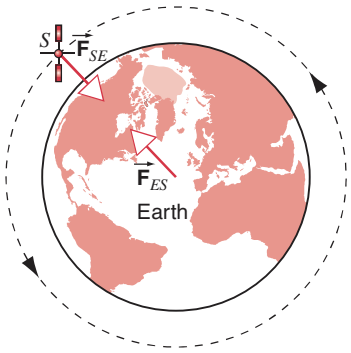

FIGURE 3-10. A satellite in Earth orbit. The forces shown are an action–reaction pair. Note that they act on different bodies.

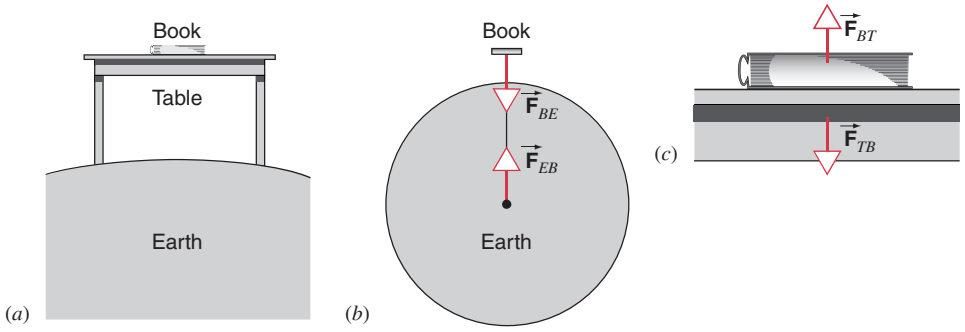

FIGURE 3-11. (*a*) A book rests on a table, which in turn rests on the Earth. (*b*) The book and the Earth exert gravitational forces on each other, forming an action–reaction pair. (*c*) The table and book exert action–reaction contact forces on each other.

force exerted *on* the satellite *by* the gravitational pull of the Earth. Where is the corresponding reaction force? It is $\vec{F}_{ES}$, the force acting on the Earth due to the gravitational pull of the satellite.

You may think that the tiny satellite cannot exert much of a gravitational pull on the Earth but it does, exactly as Newton's third law requires ($\vec{F}_{ES} = -\vec{F}_{SE}$). The force $\vec{F}_{ES}$ causes the Earth to accelerate, but, because of the Earth's large mass, its acceleration is so small that it cannot easily be detected.

2. *A book resting on a table.* Figure 3-11*a* shows a book resting on a table. The Earth pulls downward on the book with a force $\vec{F}_{BE}$. The book does not accelerate because the effect of this force is balanced by an equal and opposite contact force $\vec{F}_{BT}$ exerted on the book by the table.

Even though $\vec{F}_{BE}$ and $\vec{F}_{BT}$ are equal in magnitude and oppositely directed, they do *not* form an action–reaction pair. Why not? *Because they act on the same body—the book.* The two forces sum to zero and thus account for the fact that the book is not accelerating.

Each of these forces must then have a corresponding reaction force somewhere. Where are they? The reaction to $\vec{F}_{BE}$ is $\vec{F}_{EB}$, the (gravitational) force with which the book attracts the Earth. We show this action–reaction pair in Fig. 3-11*b*.

Figure 3-11*c* shows the reaction force to $\vec{F}_{BT}$. It is $\vec{F}_{TB}$, the contact force on the table due to the book. The action–reaction pairs involving the book in this problem, and the bodies on which they act, are

$$\text{first pair:} \quad \vec{F}_{BE} = -\vec{F}_{EB} \qquad \text{(book and Earth)}$$

and

$$\text{second pair:} \quad \vec{F}_{BT} = -\vec{F}_{TB} \qquad \text{(book and table).}$$

3. *Pushing a row of crates.* Figure 3-12 shows a worker *W* pushing two crates, each of which rests on a wheeled cart that can roll with negligible friction. The worker exerts a force $\vec{F}_{1W}$ on crate 1, which in turn pushes back on the worker with a reaction force $\vec{F}_{W1}$. Crate 1 pushes on crate 2 with a force $\vec{F}_{21}$, and crate 2 pushes back on crate 1 with a force $\vec{F}_{12}$. (Note that the worker exerts no force on crate 2 directly.) To move forward, the worker must push backward against the ground. The worker exerts a force $\vec{F}_{GW}$ on the ground, and the reaction force of the ground on the worker, $\vec{F}_{WG}$, pushes the worker forward. The figure shows three action–reaction pairs:

$$\vec{F}_{21} = -\vec{F}_{12} \qquad \text{(crate 1 and crate 2),}$$

$$\vec{F}_{1W} = -\vec{F}_{W1} \qquad \text{(worker and crate 1),}$$

$$\vec{F}_{WG} = -\vec{F}_{GW} \qquad \text{(worker and ground).}$$

Note that in this example the worker is the active agent that is responsible for the motion, but it is the reaction force on the worker by the ground that makes this possible. If there were no friction between the worker's shoes and the ground, the worker could not move the system forward.

4. *Block hanging from a spring.* Figure 3-13*a* shows a block hanging at rest from a spring, the other end of which is fixed to the ceiling. The forces on the block, shown separately in Fig. 3-13*b*, are its weight $\vec{F}_{BE}$ (the gravitational force on the block by the Earth) and the force $\vec{F}_{BS}$ exerted on the block by the spring. The block is at rest under the influence of these forces, but they are *not* an action–reaction pair, because once again they act on the same body. The reaction force to the weight $\vec{F}_{BE}$ is the gravitational force $\vec{F}_{EB}$ that the block exerts on the Earth, which is not shown.

The reaction force to $\vec{F}_{BS}$ (the force exerted *on* the block *by* the spring) is the force $\vec{F}_{SB}$ exerted *on* the spring *by* the block. To show this force, we illustrate the forces acting on the spring in Fig. 3-13*c*. These forces include the reaction to $\vec{F}_{BS}$, which we show as a force $\vec{F}_{SB}$ ($= -\vec{F}_{BS}$) acting downward, the weight $\vec{F}_{SE}$ of the spring (usually negligible), and the upward pull $\vec{F}_{SC}$ on the spring by the ceiling. If the spring is at rest, the net force must be zero: $\vec{F}_{SC} + \vec{F}_{SE} + \vec{F}_{SB} = 0$.

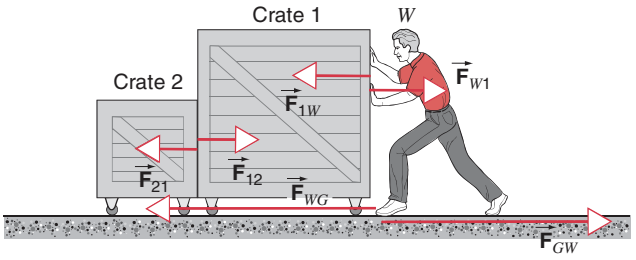

FIGURE 3-12. A worker pushes against crate 1, which in turn pushes on crate 2. The crates are on wheels that move freely, so there is no friction between the crates and the ground.

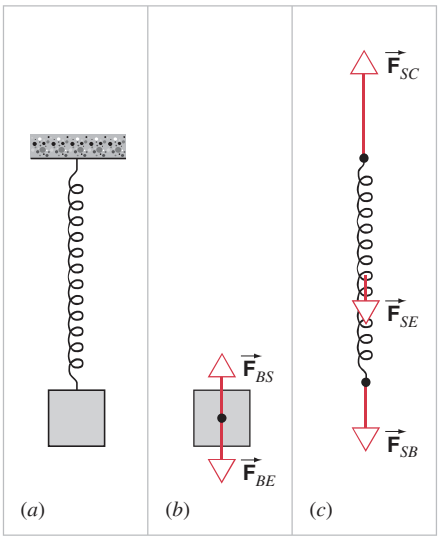

FIGURE 3-13. (*a*) A block hangs at rest supported by a stretched spring. (*b*) The forces on the block. (*c*) The forces on the spring.

The reaction force to $\vec{F}_{SC}$ acts *on* the ceiling. Since we are not showing the ceiling as an independent body in this diagram, the reaction to $\vec{F}_{SC}$ does not appear.

Verifying Newton's Third Law

We can easily verify Newton's third law by attaching electronic force probes (Fig. 3-5) to two carts that collide on a frictionless track. These force probes are connected to a computer, which plots the force instantaneously as the two carts collide.

Figure 3-14 shows the results of three different collisions between the carts. In Fig. 3-14*a*, cart 1 was originally at rest when cart 2 (of equal mass) collided with it. Note that at every instant of time, the force exerted on cart 1 by cart 2 is equal and opposite to that exerted on cart 2 by cart 1.

Figure 3-14*b* shows the results when the same two carts experience a head-on collision with both carts in motion, and Fig. 3-14*c* shows the results of a head-on collision between two carts when cart 2 has three times the mass of cart 1.

In every case, no matter which cart is in motion and no matter what the relative masses of the two carts are, the forces are exactly equal and opposite, just as Newton's third law requires.

3-7 WEIGHT AND MASS

Consider a body of mass m that is released from rest near the surface of the Earth. As we discussed in Section 2-6, the body will fall downward with the free-fall acceleration $\vec{\mathbf{g}}$ (if we neglect other forces that may be acting, such as air resistance). Here $\vec{\mathbf{g}}$ is a vector whose magnitude g is the free-fall acceleration and whose direction is vertically down (toward the center of the Earth). Assuming the Earth's surface to be an inertial frame so that we may apply Newton's second law, the net force on the object must equal $m\vec{\mathbf{a}}$, or $m\vec{\mathbf{g}}$ in our case since $\vec{\mathbf{a}} = \vec{\mathbf{g}}$. This force is due to the Earth's gravitational attraction for the body.

If we instead hold the body in our hand rather than releasing it, its acceleration is zero and hence the net vertical force must be zero (by Newton's second law). We have not "turned off" the Earth's gravitational attraction for the body; this force still acts on the body and can still be expressed as $m\vec{\mathbf{g}}$. The hand must therefore be exerting an upward force equal in magnitude to the downward force $m\vec{\mathbf{g}}$, so the net force is zero. (These two forces are *not* an action–reaction pair, because they act on the same body.) We sense the need to exert this upward force by the tension in our muscles; in this way we can "feel" the Earth's gravitational attraction for the body.

The downward force of the Earth's gravity acting on the body is called the *weight* of the body. The force of the Earth's gravity on the body is the same whether it is at rest or falling; the force has magnitude mg and a direction toward the center of the Earth. In terms of magnitudes, the weight W is

$$W = mg. \tag{3-7}$$

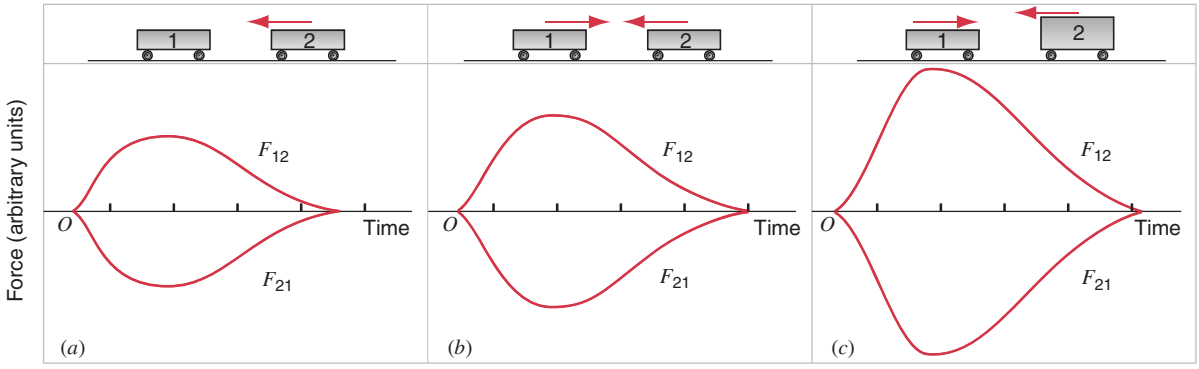

FIGURE 3-14. Electronic force probes of the type shown in Fig. 3-5 are mounted on two colliding carts. The probes simultaneously measure the force exerted on each cart by the other. The plots show the forces exerted on the two carts as functions of the time during the collisions. (*a*) Cart 2 is initially in motion and collides with cart 1, which is initially at rest. The two carts have equal masses. (*b*) A head-on collision between two carts of equal masses. (*c*) A head-on collision between two carts in which cart 2 has three times the mass of cart 1.

Weight is measured in force units, such as newtons or pounds.

We can measure the weight directly if we place the body on a platform scale (such as a bathroom scale) with a display that indicates the magnitude of the force that the platform is exerting on the body; if the body is at rest, then the net vertical force is zero, and so the upward force on the body due to the platform must equal the downward force on the body due to the Earth (the weight). We can also measure this force using a spring scale, such as might be found in the produce section of a supermarket; again, the net force on the body must be zero, and the spring exerts an upward force (which can be read on the scale) equal in magnitude to the downward force mg.

When you draw a free-body diagram for a body near the Earth's surface, you should include a force $m\vec{\mathbf{g}}$ directed toward the center of the Earth. This represents the weight, the gravitational force on the body due to the Earth. The third-law reaction force to the weight is the gravitational force on the Earth due to the body; this force would appear only on a free-body diagram of the Earth (as in Figs. 3-10 or 3-11*b*).

In this discussion, we have assumed that the surface of the Earth is an inertial frame. This is only approximately true; because of the Earth's rotation its surface is not an inertial frame, but the error resulting from this assumption is very small—about 0.3% at the equator, where the effect is largest. That is, the weight indicated on a scale at the equator is 0.3% smaller than the force of gravity on the body. At the poles, the scale reading and the free-fall acceleration are not affected by the rotation. Neglecting this effect, we can regard the scale readings at the Earth's surface as a sufficiently precise measure of the weight of an object.

The Difference between Weight and Mass

As Eq. 3-7 shows, the weight depends on the mass—the greater the mass, the greater the weight. A second body with twice the mass of the first will have twice the weight at the same location. However, weight and mass are very different quantities. Our definition of mass in Section 3-4 and our operational procedure for measuring it make no reference to the gravitational force of attraction by the Earth. We could use the same procedure, and arrive at the same values of the mass in comparison with the standard kilogram, if we carried out the measurements on the Moon (where the free-fall acceleration is only $\frac{1}{6}$ the value on Earth) or even in empty space far from any planet or star (where the free-fall acceleration is zero). The weight of the body would be different in these locations, but the mass is the same; that is, we must apply the same amount of force at each location to produce a given acceleration.

The mass of a body has the same value in any location, but its weight will vary on the surface of the Earth where the free-fall acceleration varies with location. At a location on the Earth's surface where $g = 9.78\ \text{m/s}^2$ (near the equator, for example) a body of mass 1.00 kg has a weight of 9.78 N, while near the poles where $g = 9.83\ \text{m/s}^2$ (because the poles are closer to the center of the Earth than the equator) that same body has a weight of 9.83 N. Identical spring scales in the two locations would stretch by slightly different amounts to record these different weights. Unlike the mass, which is an *intrinsic* property of the body, the weight of a body depends on its location relative to the center of the Earth. (Variations in g on Earth are discussed in Section 14-4.)

Equation 3-7 shows that, for a given value of g, mass and weight are proportional to each other. Sometimes you may see an equation in which mass units are set equal to weight units—for example, 1 kg = 2.2 lb. This equation violates the rules for dimensional consistency we discussed in Section 1-7. Here the equal sign means "is equivalent to," and the equation should be taken to mean that at a location where g has a certain value, an object of mass 1 kg is equivalent to a weight of 2.2 lb. On the surface of the Moon, this equation would read 1 kg = 0.37 lb, but at the surface of Jupiter 1 kg = 5.1 lb. A "quarter-pounder" hamburger at a fast-food outlet on Jupiter would contain about 14 times as much meat as one on the Moon, but a 0.1-kg hamburger (about $\frac{1}{4}$ lb on Earth) would contain exactly the same amount of meat at all locations. When we colonize the planets, we should be sure to order supplies by mass, not by weight!

Weightlessness

Photographs of astronauts in orbiting space vehicles (such as Fig. 3-15) show them floating freely in a state that we call "weightless." We must be careful how we analyze the astronauts' motion, because in their rapidly orbiting craft they are not even approximately in an inertial frame. An object released from rest, according to the astronaut's noninertial reference frame, stays at the same location. Thus the free-fall acceleration appears to be zero in that frame. However, an object released from a nonorbiting craft at that same altitude (about 400 km in the case of the space shut-

FIGURE 3-15. Astronaut Dr. Mae C. Jemison in free-fall in the orbiting space shuttle *Endeavor* appears to float as if weightless.

FIGURE 3-16. Actors in simulated weightlessness during the filming of the movie *Apollo 13*. They are in free-fall in a KC-135 aircraft flying a parabolic trajectory. This aircraft, nicknamed the "Vomit Comet," is used by NASA for microgravity research.

tle) would fall toward the center of the Earth with a free-fall acceleration of about 8.7 m/s^2.

If we were to place a body on a platform scale or attach it to a spring scale in the astronaut's noninertial reference frame, the scale would read zero. In this reference frame, we cannot use the scale reading to determine the weight of the body. However, the body is certainly *not* "massless," nor is it "gravityless." With $g = 8.7$ m/s^2 at that altitude, a 1-kg body would have a weight of 8.7 N, about 11% less than its weight on the Earth's surface.

Our perception or sensation of weight involves the force with which the floor pushes up on us. Floating in water, we are less aware of our *weight,* but we are fully aware of our *mass,* such as when we try to accelerate by swimming through the water. If we stand in an elevator that is accelerating upward, the floor exerts a force on us that is greater than the pull of gravity and so we feel as if our weight increases; when the elevator accelerates downward, we feel as if our weight has become smaller. If we stand on a platform scale in an accelerating elevator, the scale readings will confirm these perceptions (see Sample Problem 3-7). However, the magnitude of the weight remains mg, independent of any acceleration.

True weightlessness can be achieved only in deep space, far from any star or planet. In a spacecraft drifting with its engines off, the astronauts would float freely. If the engines fire, the resulting acceleration would cause the ship to be a noninertial frame; in the astronauts' frame of reference, the floor of the accelerating ship would exert an upward force that would be perceived by the astronauts as similar to weight. In a similar way, if the ship rotates, the outer wall of the ship would be the floor that provides the sensation of weight by pushing anything in contact with the floor toward the axis of rotation. This effect is sometimes referred to as "artificial gravity" and will be used in the International Space Station to provide the sensation of weight for biological specimens.

A body in free fall near the Earth's surface has no floor to push on it and therefore would feel weightless. If you were inside a chamber that is also in free fall (such as an elevator cab in which the cable breaks), the floor would not push on you and you would feel no sensation of weight. As we will learn in the next chapter, a projectile in free fall near the Earth's surface follows a parabolic trajectory; if an airplane flies a chosen parabolic trajectory, the passengers inside will be objects in free fall and will feel weightless because they are not in contact with the floor of the plane. This effect is used to train astronauts for working in the similar free fall of Earth orbit and has also been used in movies to simulate the effects of orbit (Fig. 3-16). Even though a body in free fall near the Earth's surface lacks the sensation of weight that ordinarily comes from the upward push of a floor, the weight remains at the value mg, indicating the strength of the Earth's gravitational attraction for the body.

3-8 APPLICATIONS OF NEWTON'S LAWS IN ONE DIMENSION

Although each problem to be solved with Newton's laws will require a unique approach, the general procedure that we gave in Section 3-5 forms the basis for the analysis of all such problems. The best way to learn the applications of

the rules is to study the examples. Often in these problems there are two or more bodies to which Newton's laws must be separately applied.

In these problems we make some assumptions that simplify the analysis at the cost of some physical reality. Bodies are treated as particles, so that all forces can be considered to act at a single point. String are massless (no force is required to accelerate them) and inextensible (they do not stretch, so that bodies connected by taut strings have the same speed and acceleration). Despite these simplifications, the examples provide insight into the basic techniques of dynamical analysis. Later in the text we add new techniques that permit us to be more realistic in our analysis. For now, we ignore many admittedly important effects so that we can concentrate on the basic methods used to solve problems.

SAMPLE PROBLEM 3-4. A worker W is pushing a packing crate of mass $m_1 = 4.2$ kg. In front of the crate is a second crate of mass $m_2 = 1.4$ kg (Fig. 3-17*a*). Both crates slide across the floor without friction. The worker pushes on crate 1 with a force $P_{1W} = 3.0$ N. Find the accelerations of the crates and the force exerted by crate 1 on crate 2.

Solution We choose the positive x axis to be in the direction of motion of the crates, so that force and acceleration components to the right in Fig. 3-17 are positive. The worker pushes only on crate 1. The force $\vec{\mathbf{F}}_{21}$ pushing on crate 2 is exerted on it by crate 1. According to Newton's third law, crate 2 then exerts a force $\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$ on crate 1. With F_{12} and F_{21} representing the magnitudes of these forces, the x component of $\vec{\mathbf{F}}_{12}$ is $-F_{12}$ and that of $\vec{\mathbf{F}}_{21}$ is F_{21}. Figures 3-17*b*, *c* show the free-body diagrams of crates 1 and 2. The net force on m_1 is $\Sigma F_x = P_{1W} - F_{12}$, and for crate 1 Newton's second law ($\Sigma F_x = ma_x$) then gives

$$\text{(crate 1)} \qquad P_{1W} - F_{12} = m_1 a_1,$$

where a_1 represents the x component of the acceleration of crate 1. The net force on crate 2 is $\Sigma F_x = F_{21}$, so Newton's second law gives

$$\text{(crate 2)} \qquad F_{21} = m_2 a_2.$$

If the two crates stay in contact, then $a_1 = a_2$. We call this common acceleration a. Adding these equations, we obtain

$$P_{1W} - F_{12} + F_{21} = m_1 a + m_2 a$$

or, using Newton's third law for the magnitudes of the contact forces ($F_{12} = F_{21}$) and solving for a,

$$a = \frac{P_{1W}}{m_1 + m_2} = \frac{3.0 \text{ N}}{4.2 \text{ kg} + 1.4 \text{ kg}} = 0.54 \text{ m/s}^2.$$

It should not be too surprising that the acceleration is determined by the total mass $m_1 + m_2$ of the system of two crates, because the force P_{1W} exerted by the worker is ultimately responsible for accelerating the entire system. To find the contact force exerted on crate 2 by crate 1, we have

$$F_{21} = m_2 a = (1.4 \text{ kg})(0.54 \text{ m/s}^2) = 0.76 \text{ N}.$$

Note that the force exerted on crate 2 by crate 1 (equal to 0.76 N) is smaller than the force exerted on crate 1 by the worker (3.0 N). This is reasonable, because F_{21} acts only to accelerate crate 2, but P_{1W} acts to give the same acceleration to both crates.

SAMPLE PROBLEM 3-5. A flat-bed cart of mass $m_C = 360$ kg rolls on frictionless wheels. Resting on the cart is a box of mass $m_B = 150$ kg (Fig. 3-18*a*). The box can slide on the cart, but each exerts a force (due to friction) on the other during the sliding. When a worker pulls on the box with a force $\vec{\mathbf{F}}_{BW}$, both the box and the cart move forward, but the box moves faster than the cart, because the frictional force is not strong enough to prevent the box from sliding forward on the cart. An observer measures the magnitudes of the accelerations to be 1.00 m/s^2 for the box and 0.167 m/s^2 for the cart. Find (*a*) the frictional force between the box and the cart and (*b*) the force that the worker is exerting on the box.

Solution (*a*) We choose the x axis so that its positive direction is to the right in Fig. 3-18. Force and acceleration components in that direction are positive. The worker exerts a force $\vec{\mathbf{F}}_{BW}$ on the box. It is customary to use a lower-case symbol $\vec{\mathbf{f}}$ to represent frictional forces, so the force on the box due to the cart is $\vec{\mathbf{f}}_{BC}$, which points to the left (opposing the motion of the box) and has x component $-f_{BC}$. By Newton's third law, there is an equal and opposite frictional force $\vec{\mathbf{f}}_{CB}$ on the cart exerted by the box. Figures 3-18*b*, *c* show the free-body diagrams of the cart and the box.

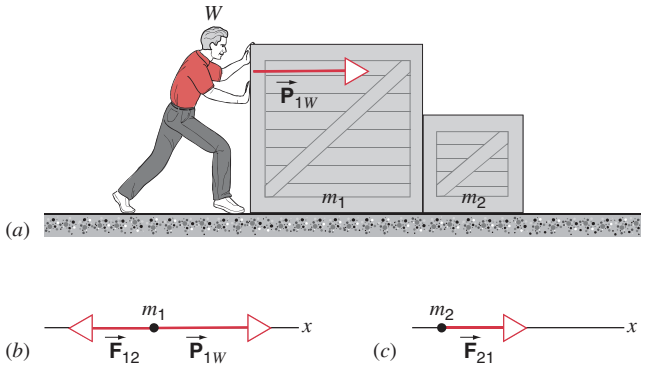

FIGURE 3-17. Sample Problem 3-4. (*a*) A worker pushes on a crate, which in turn pushes on another crate. (*b*) The free-body diagram of crate 1. (*c*) The free-body diagram of crate 2.

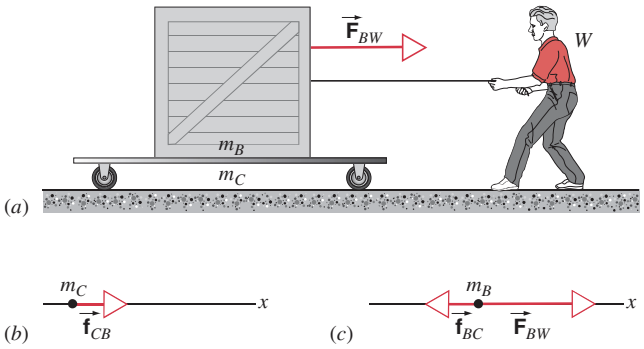

FIGURE 3-18. Sample Problems 3-5 and 3-6. (*a*) A worker pulls on a box that slides on a rolling cart. (*b*) The free-body diagram of the cart. (*c*) The free-body diagram of the box.

The net force on the cart is $\Sigma F_x = f_{CB}$, and applying Newton's second law ($\Sigma F_x = ma_x$) to the cart, we obtain

(cart) $$f_{CB} = m_C a_C = (360\text{ kg})(0.167\text{ m/s}^2) = 60\text{ N}.$$

(*b*) Similarly, the net force on the box is $\Sigma F_x = F_{BW} - f_{BC}$, so Newton's second law gives

(box) $$F_{BW} - f_{BC} = m_B a_B$$

and solving for F_{BW} we obtain

$$F_{BW} = f_{BC} + m_B a_B = 60\text{ N} + (150\text{ kg})(1.00\text{ m/s}^s) = 210\text{ N},$$

where we have used $f_{BC} = f_{CB}$ for the magnitudes of the frictional forces, which form an action–reaction pair.

Friction with the box pulls the cart forward. Even though in this case friction produces the motion of the cart, friction between two objects always opposes their *relative* motion. If there were no friction in this problem, the cart would not move at all and there would be more relative motion between the box and the cart. If the frictional force were sufficiently large (see the next Sample Problem), the box and the cart could move together with *no* relative motion between them.

SAMPLE PROBLEM 3-6. Suppose in the previous Sample Problem that the frictional force were larger, so that the box does not slide on the cart (the box and the cart move together as a unit). If the force applied by the worker remains the same at 210 N, what is the frictional force of the box on the cart?

Solution The cart and the box move together, so they have the same acceleration a. Newton's second law gives

(cart) $$\Sigma F_x = f_{CB} = m_C a$$

(box) $$\Sigma F_x = F_{BW} - f_{BC} = m_B a.$$

Since we want to solve for the unknown $f_{CB} = f_{BC}$, we eliminate a from these two equations and obtain

$$f_{CB} = \frac{m_C F_{BW}}{m_C + m_B} = \frac{(360\text{ kg})(210\text{ N})}{360\text{ kg} + 150\text{ kg}} = 150\text{ N}.$$

SAMPLE PROBLEM 3-7. A passenger P of mass $m = 72.2$ kg is riding in an elevator while standing on a platform scale (Fig. 3-19*a*, which is essentially a calibrated spring scale that reads the upward force F_{PS} exerted on the passenger by the scale. (The situation would be exactly the same if the passenger were hanging from a spring scale.) What does the scale read when the elevator cab is (*a*) descending with constant velocity and (*b*) ascending with an acceleration of 3.20 m/s^2?

Solution (*a*) First we develop a general result that is valid for any acceleration a. We choose our inertial reference frame to be that of the building in which the elevator is located, because an accelerating elevator is not an inertial reference frame. Both g and a are measured by an observer in this inertial frame. We choose our coordinate system so that the y axis is vertical and positive upward. Figure 3-19*b* shows the free-body diagram of the passenger. Two forces act on the passenger: the upward force $\vec{\mathbf{F}}_{PS}$ exerted by the scale and the downward weight $\vec{\mathbf{W}}$ of magnitude $W = mg$ (the force of the Earth's gravity on the passenger).

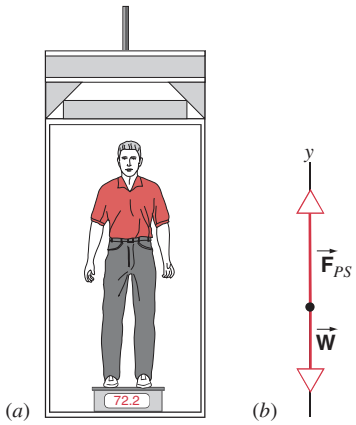

FIGURE 3-19. Sample Problem 3-7. (*a*) A passenger is riding in an elevator cab while standing on a scale. Like most such scales, this one reads in mass units (kilograms), rather than in the corresponding force units (newtons). (*b*) The free-body diagram of the passenger.

The net force on the passenger is then $\Sigma F_y = F_{PS} - W$, and Newton's second law ($\Sigma F_y = ma_y$) gives

$$F_{PS} - W = ma_y$$

or

$$F_{PS} = W + ma_y = mg + ma_y.$$

When $a_y = 0$ (corresponding to motion with constant velocity) then

$$F_{PS} = mg = (72.2\text{ kg})(9.80\text{ m/s}^2) = 708\text{ N}\ (= 159\text{ lb}).$$

The scale reading does not depend on the velocity of the elevator, and the scale reads the same when the elevator moves with constant velocity as it does when the elevator is at rest.

(*b*) When $a_y = +3.20$ m/s^2, we have

$$F_{PS} = m(g + a_y) = (72.2\text{ kg})(9.80\text{ m/s}^2 + 3.20\text{ m/s}^2)$$
$$= 939\text{ N}\ (= 211\text{ lb}).$$

The scale reading increases when the elevator is accelerating upward and decreases when the elevator is accelerating downward. What does the scale read when the elevator is moving upward but accelerating downward (that is, it is slowing down)? What does it read when the cable breaks and the elevator is in free fall ($a_y = -g$)?

MULTIPLE CHOICE

3-1 Classical Mechanics

3-2 Newton's First Law

1. An interstellar spacecraft, far from the influence of any stars or planets, is moving at high speed under the influence of fusion rockets when the engines malfunction and stop. The spacecraft will
(A) immediately stop, throwing all of the occupants to the front of the craft.
(B) begin slowing down, eventually coming to a rest in the cold emptiness of space.
(C) keep moving at constant speed for a while, but then begin to slow down.
(D) keep moving forever at the same speed.

2. A small child is playing with a ball on a level surface. She gives the ball a push to get it rolling, then the ball rolls a short distance before coming to a stop. The ball slows to a stop because
(A) the child stopped pushing it.
(B) speed is proportional to force.
(C) there must have been some force on the ball opposing the direction of motion.
(D) the net force on the ball was zero, so it wanted to remain stationary.

3-3 Force

3. A student attaches a ruler to a block of wood sitting on a horizontal surface as shown in Fig. 3-20*a*. The surface exerts a large frictional force on the block. A nail is fastened at the 0-in. mark. The student pulls the rubber band tight at the 5-in. mark; he then pulls it tighter and when it reaches the 8-in. mark, the block of wood just starts to move (Fig. 3-20*b*). Before the block begins to move, the net force on the block when the rubber band is stretched 7 in. is
(A) greater than (B) equal to
(C) less than (D) unrelated to
the net force on the block when the rubber band is stretched 6 in.

4. The student in multiple-choice question 3 then pulls the rubber band so that it is at the 9-in. mark. The block of wood slides faster and faster with a constant acceleration, and as it moves, the student moves his hand so that the rubber band is always pulled to the 9-in. mark. The net force on the block when the rubber band is stretched 9 in. is
(A) greater than (B) equal to
(C) less than (D) unrelated to
the net force on the block when the rubber band was stretched 7 in.

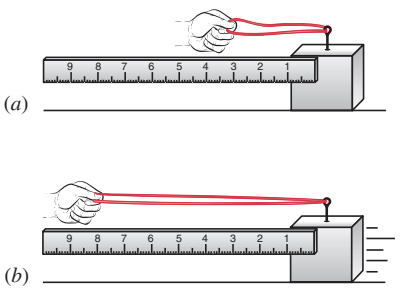

FIGURE 3-20. Multiple-choice questions 3 and 4.

3-4 Mass

5. Two objects with masses M and m ($M > m$) are on a frictionless surface. A force F will accelerate the smaller object with an acceleration a. If that same force is applied to the larger object then it will
(A) move with a greater acceleration.
(B) move with the same acceleration.
(C) move but with a smaller acceleration.
(D) move only if the force F is greater than some minimum value.

3-5 Newton's Second Law

6. An object is moving north. From only this information one can conclude
(A) that there is a single force on the object directed north.
(B) that there is a net force on the object directed north.
(C) that there may be several forces on the object, but the largest must be directed north.
(D) nothing about the forces on the object.

7. An object is moving north with an increasing speed. From only this information one can conclude
(A) that there is a single force on the object directed north.
(B) that there is a net force on the object directed north.
(C) that there may be several forces on the object, but the largest must be directed north.
(D) nothing about the forces on the object.

8. Which of the following objects is not experiencing a net force directed north?
(A) An object moving south at a decreasing speed.
(B) An object moving north at an increasing speed.
(C) An object instantaneously at rest that then begins to move north.
(D) An object moving north at a constant speed.

3-6 Newton's Third Law

9. A rock rests on a level surface. The magnitude of the force on the surface due to the rock is F_{SR}, and the magnitude of the force on the rock due to the surface is F_{RS}. If we compare these forces, we find
(A) $F_{SR} < F_{RS}$. (B) $F_{SR} = F_{RS}$. (C) $F_{SR} > F_{RS}$.
(D) There is not enough information to compare the two forces.

10. A rock is on an inclined surface. The rock is originally at rest, but starts to slide down the incline. The magnitude of the force on the surface due to the rock is F_{SR}, and the magnitude of the force on the rock due to the surface is F_{RS}. If we compare these forces, we find
(A) $F_{SR} < F_{RS}$ always.
(B) $F_{SR} = F_{RS}$ when the rock is at rest but then $F_{SR} > F_{RS}$.
(C) $F_{SR} = F_{RS}$ always.
(D) $F_{SR} > F_{RS}$ always.

11. A piano is rolling down a frictionless slope at an ever increasing speed. The piano tuner sees it, runs up to it and pushes on it, slowing it down to a constant speed. The magnitude of the force on the man by the piano is F_{MP}; the magnitude of the force on the piano by the man is F_{PM}. If we compare these forces, we find
(A) $F_{PM} > F_{MP}$ always.

(B) $F_{PM} > F_{MP}$ while the piano slows down but $F_{PM} = F_{MP}$ when the piano is moving at constant speed.
(C) $F_{PM} = F_{MP}$ always.
(D) $F_{PM} = F_{MP}$ while the piano slows down but $F_{PM} < F_{MP}$ when the piano is moving at constant speed.

3-7 Weight and Mass

12. A large rock *falls* on your toe. Which of the concepts is most important in determining how much it hurts?
(A) The mass of the rock.
(B) The weight of the rock.
(C) Both the mass and the weight of the rock are important.
(D) Either the mass or the weight, as the two are related by a single multiplicative constant g.

13. A large rock *sits* on your toe. Which of the concepts is most important in determining how much it hurts?
(A) The mass of the rock.
(B) The weight of the rock.
(C) Both the mass and the weight of the rock are important.
(D) Either the mass or the weight, as the two are related by a single multiplicative constant g.

3-8 Applications of Newton's Laws in One Dimension

14. An object is free to move on a table, except that there is a constant frictional force f that opposes the motion of the object when it moves. If a force of 10.0 N pulls the object, the acceleration is 2.0 m/s^2. If a force of 20.0 N pulls the object, the acceleration is 6.0 m/s^2.
(a) What is the force of friction f?
(A) 1.0 N. (B) 3.33 N. (C) 5.0 N. (D) 10.0 N.
(b) What is the mass of the object?
(A) 0.40 kg. (B) 2.5 kg. (C) 3.33 kg. (D) 5.0 kg.

15. A parachutist is in free fall before opening her chute. The net force on her has magnitude F and is directed down; this net force is somewhat less than her weight W because of air friction. Then she opens her chute. At the instant after her chute fully inflates the net force on her is
(A) greater than F and still directed down.
(B) less than F and still directed down.
(C) zero.
(D) directed upward, but could be more or less than F.

16. (a) You stand on a spring-loaded bathroom scale in a bathroom. The scale "reads" your mass. What is the scale actually *measuring?* (b) You stand on a spring-loaded bathroom scale in an elevator that is accelerating upward at 2.0 m/s^2. The scale "reads" your mass. What is the scale *measuring?*
(A) Your mass.
(B) Your weight.
(C) The force of the scale pushing up on your feet.
(D) The force of your feet pushing down on the scale.

QUESTIONS

1. Of the objects listed in Table 3-1, which might be considered as a particle for the motion described? For those that behave like particles, can you describe a type of motion in which they could *not* be considered as particles?

2. Why do you fall forward when a moving bus decelerates to a stop and fall backward when it accelerates from rest? Subway standees often find it convenient to face the side of the car when the train is starting or stopping and to face the front or rear when it is running at constant speed. Why?

3. Why did we specify the use of a "light" spring for the experiments described in Section 3-3? What would be the difference if we used a "heavy" spring?

4. A block with mass m is supported by a cord C from the ceiling, and a similar cord D is attached to the bottom of the block (Fig. 3-21). Explain this: If you give a sudden jerk to D, it will break, but if you pull on D steadily, C will break.

5. Criticize the statement, often made, that the mass of a body is a measure of the "quantity of matter" in it.

6. Using force, length, and time as fundamental quantities, what are the dimensions of mass?

7. How many slugs are in one kilogram?

8. A car moving at constant speed is suddenly braked. The occupants, all wearing seat belts, are thrown forward. The instant the car stops, however, the occupants are all jerked backward. Why? Is it possible to stop an automobile without this "jerk"?

9. Can Newton's first law be considered merely a special case of the second law with $\vec{\mathbf{a}} = 0$? If so, is the first law really needed? Discuss.

10. What is the relation—if any—between the force acting on an object and the direction in which the object is moving?

11. Suppose a body that is acted on by exactly two forces is accelerated. Does it then follow that (a) the body cannot move with constant speed; (b) the velocity can never be zero; (c) the sum of the two forces cannot be zero; (d) the two forces must act in the same line?

12. A horse is urged to pull a wagon. The horse refused to try, citing Newton's third law as a defense: the pull of the horse on the wagon is equal but opposite to the pull of the wagon on the horse. "If I can never exert a greater force on the wagon than it exerts on me, how can I ever start the wagon moving?" asks the horse. How would you reply?

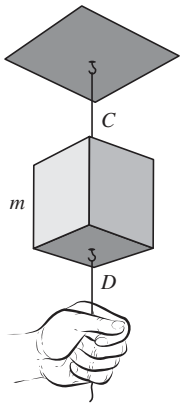

FIGURE 3-21. Question 4.

13. Comment on whether the following pairs of forces are examples of action–reaction: (*a*) The Earth attracts a brick; the brick attracts the Earth. (*b*) A propellered airplane pushes air toward the tail; the air pushes the plane forward. (*c*) A horse pulls forward on a cart, moving it; the cart pulls backward on the horse. (*d*) A horse pulls forward on a cart without moving it; the cart pulls back on the horse. (*e*) A horse pulls forward on a cart without moving it; the Earth exerts an equal and opposite force on the cart. (*f*) The Earth pulls down on the cart; the ground pushes up on the cart with an equal and opposite force.

14. The following statement is true; explain it. Two teams are having a tug of war; the team that pushes harder (horizontally) against the ground wins.

15. Two students try to break a rope. First they pull against each other and fail. Then they tie one end to a wall and pull together. Is this procedure better than the first? Explain your answer.

16. What is your mass in slugs? Your weight in newtons?

17. A French citizen, filling out a form, writes "78 kg" in the space marked Poids (weight). However, weight is a force and the kilogram is a mass unit. What do the French (among others) have in mind when they use a mass unit to report their weight? Why don't they report their weight in newtons? How many newtons does this person weigh? How many pounds?

18. Comment on the following statements about mass and weight taken from examination papers. (*a*) Mass and weight are the same physical quantities expressed in different units. (*b*) Mass is a property of one object alone, whereas weight results from the interaction of two objects. (*c*) The weight of an object is proportional to its mass. (*d*) The mass of a body varies with changes in its local weight.

19. A horizontal force acts on a body that is free to move. Can it produce an acceleration if the force is less than the weight of that body?

20. Why does the acceleration of a freely falling object not depend on the weight of the object?

21. Describe several ways in which you could, even briefly, experience weightlessness.

22. Under what circumstances would your weight be zero? Does your answer depend on the choice of a reference system?

23. The "mechanical arm" on the space shuttle can handle a 2200-kg satellite when extended to 12 m; see Fig. 3-22. Yet, on the ground, this remote manipulator system (RMS) cannot support its own weight. In the "weightlessness" of an orbiting shuttle, why does the RMS have to be able to exert any force at all?

24. In November 1984, astronauts Joe Allen and Dale Gardner salvaged a Westar-6 communications satellite from a faulty orbit and placed it into the cargo bay of the space shuttle *Discovery;* see Fig. 3-22. Describing the experience, Joe Allen said of the satellite, "It's not heavy; it's massive." What did he mean?

25. The owner's manual of a car suggests that your seat belt should be adjusted "to fit snugly" and that the front seat head rest should not be adjusted so that it fits comfortably at the back of your neck but so that "the top of the head rest is level with the top of your ears." How do Newton's laws support these good recommendations?

FIGURE 3-22. Questions 23 and 24.

26. Is it possible to derive $\Sigma \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ from some other principle? Is $\Sigma \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ an experimental conclusion?

27. Observers in two different inertial frames will measure the same acceleration of a moving object. Will they measure the same velocity of a moving object? Will they measure the same force on the moving object?

28. You are an astronaut in the lounge of an orbiting space station and you remove the cover from a long thin jar containing a single olive. Describe several ways—all taking advantage of the inertia of either the olive or the jar—to remove the olive from the jar.

29. In Fig. 3-23, a needle has been placed in each end of a broomstick, the tips of the needles resting on the edges of filled wine glasses. The experimenter strikes the broomstick a swift and sturdy blow with a stout rod. The broomstick breaks and falls to the floor but the wine glasses remain in place and no wine is spilled. This impressive parlor stunt was popular at the end of the nineteenth century. What is the physics behind it? (If you try it, practice first with empty soft drink cans. Come to think of it, you might ask your physics instructor to do it, as a lecture demonstration!)

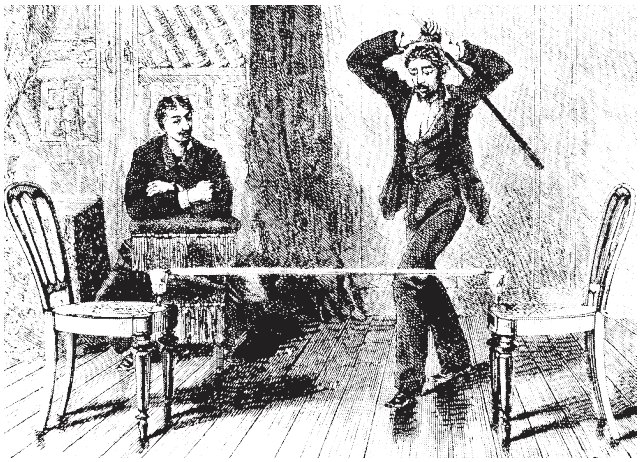

FIGURE 3-23. Question 29.

30. An elevator is supported by a single cable. There is no counterweight. The elevator receives passengers at the ground floor and takes them to the top floor, where they disembark.

New passengers enter and are taken down to the ground floor. During this round trip, when is the force exerted by the cable on the elevator equal to the weight of the elevator plus passengers? Greater? Less?

31. You are on the flight deck of the orbiting space shuttle *Discovery* and someone hands you two wooden balls, outwardly identical. One, however, has a lead core but the other does not. Describe several ways of telling them apart.

32. You stand on the large platform of a spring scale and note your weight. You then take a step on this platform and note that the scale reads less than your weight at the beginning of the step and more than your weight at the end of the step. Explain.

33. Could you weigh yourself on a scale whose maximum reading is less than your weight? If so, how?

34. A weight hangs by a spring scale from the ceiling of an elevator. In which of the following cases will the reading of the spring scale be greatest: (*a*) elevator at rest; (*b*) elevator rising with uniform speed; (*c*) elevator descending with decreasing speed; (*d*) elevator descending with increasing speed? In which will it be the least?

35. A woman stands on a spring scale in an elevator. In which of the following cases will the scale record the minimum reading: (*a*) elevator stationary; (*b*) elevator cable breaks, free fall; (*c*) elevator accelerating upward; (*d*) elevator accelerating downward; (*e*) elevator moving at constant velocity? In which will it record the maximum reading?

36. Figure 3-24 shows comet Kohoutek as it appeared in 1973. Like all comets, it moves around the Sun under the influence of the gravitational pull that the Sun exerts on it. The nucleus of the comet is a relatively massive core at a position indicated by *P*. The tail of a comet is produced by the action of the solar wind, which consists of charged particles streaming outward from the Sun. By inspection, what, if anything, can you say about the direction of the force that acts on the nucleus of the comet? What about the direction in which the nucleus is being accelerated? What about the direction in which the comet is moving?

37. In general (see Fig. 3-24), comets have a dust tail, consisting of dust particles pushed away from the Sun by the pressure of sunlight. Why is this tail often curved?

FIGURE 3-24. Questions 36 and 37.

38. Can you think of physical phenomena involving the Earth in which the Earth cannot be treated as a particle?

39. Consider a jump off a high dive. While waiting to get the courage to jump, your acceleration is zero, and you "feel" the force of gravity. When you jump you accelerate toward the water, but during this "free fall" you feel weightless, as if there was no force of gravity. Does this contradict Newton's laws of motion? How could you explain this to a non-physics student?

EXERCISES

3-1 Classical Mechanics

3-2 Newton's First Law

3-3 Force

3-4 Mass

3-5 Newton's Second Law

1. Suppose that the Sun's gravitational force was suddenly turned off, so that Earth became a free object rather than being confined to orbit the Sun. How long would it take for Earth to reach a distance from the Sun equal to Pluto's present orbital radius? (Hint: You will find some of the data you need in Appendix C.)

2. A 5.5-kg block is initially at rest on a frictionless horizontal surface. It is pulled with a constant horizontal force of 3.8 N. (*a*) What is its acceleration? (*b*) How long must it be pulled before its speed is 5.2 m/s? (*c*) How far does it move in this time?

3. An electron travels in a straight line from the cathode of a vacuum tube to its anode, which is 1.5 cm away. It starts with zero speed and reaches the anode with a speed of 5.8×10^6 m/s. Assume constant acceleration and compute the force on the electron. This force is electrical in origin. The electron's mass is 9.11×10^{-31} kg.

4. A neutron travels at a speed of 1.4×10^7 m/s. Nuclear forces are of very short range, being essentially zero outside a nucleus but very strong inside. If the neutron is captured and brought to rest by a nucleus whose diameter is 1.0×10^{-14} m, what is the minimum magnitude of the force, presumed to be constant, that acts on this neutron? The neutron's mass is 1.67×10^{-27} kg.

5. In a modified tug-of-war game, two people pull in opposite directions, not on a rope, but on a 25-kg sled resting on an icy road. If the participants exert forces of 90 N and 92 N, what is the acceleration of the sled?

6. A car traveling at 53 km/h hits a bridge abutment. A passenger in the car moves forward a distance of 65 cm (with respect to the road) while being brought to rest by an inflated air bag. What force (assumed constant) acts on the passenger's upper torso, which has a mass of 39 kg?

7. An electron is projected horizontally at a speed of 1.2×10^7 m/s into an electric field that exerts a constant vertical force of 4.5×10^{-16} N on it. The mass of the electron is

9.11×10^{-31} kg. Determine the vertical distance the electron is deflected during the time it has moved forward 33 mm horizontally.

8. The Sun yacht *Diana,* designed to navigate in the solar system using the pressure of sunlight, has a sail area of 3.1 km^2 and a mass of 930 kg. Near Earth's orbit, the Sun could exert a radiation force of 29 N on its sail. (*a*) What acceleration would such a force impart to the craft? (*b*) A small acceleration can produce large effects if it acts steadily for a long enough time. Starting from rest then, how far would the craft have moved after 1 day under these conditions? (*c*) What would then be its speed? (See "The Wind from the Sun," a fascinating science fiction account by Arthur C. Clarke of a Sun yacht race.)

9. A certain force gives object m_1 an acceleration of 12.0 m/s^2. The same force gives object m_2 an acceleration of 3.30 m/s^2. What acceleration would the force give to an object whose mass is (*a*) the difference between m_1 and m_2 and (*b*) the sum of m_1 and m_2?

10. (*a*) Neglecting gravitational forces, what force would be required to accelerate a 1200-metric-ton spaceship from rest to one-tenth the speed of light in 3 days? In 2 months? (One metric ton = 1000 kg.) (*b*) Assuming that the engines are shut down when this speed is reached, what would be the time required to complete a 5-light-month journey for each of these two cases? (Use 1 month = 30 days.)

3-6 Newton's Third Law

11. Two blocks, with masses $m_1 = 4.6$ kg and $m_2 = 3.8$ kg, are connected by a light spring on a horizontal frictionless table. At a certain instant, when m_2 has an acceleration $a_2 = 2.6$ m/s^2, (*a*) what is the force on m_2 and (*b*) what is the acceleration of m_1?

3-7 Weight and Mass

12. What are the weight in newtons and the mass in kilograms of (*a*) a 5.00-lb bag of sugar, (*b*) a 240-lb fullback, and (*c*) a 1.80-ton car? (1 ton = 2000 lb.)

13. What are the mass and weight of (*a*) a 1420-lb snowmobile and (*b*) a 412-kg heat pump?

14. A space traveler whose mass is 75.0 kg leaves Earth. Compute his weight (*a*) on Earth, (*b*) on Mars, where $g = 3.72$ m/s^2, and (*c*) in interplanetary space. (*d*) What is his mass at each of these locations?

15. A certain particle has a weight of 26.0 N at a point where the acceleration due to gravity is 9.80 m/s^2. (*a*) What are the weight and mass of the particle at a point where the acceleration due to gravity is 4.60 m/s^2? (*b*) What are the weight and mass of the particle if it is moved to a point in space where the gravitational force is zero?

16. A 12,000-kg airplane is in level flight at a speed of 870 km/h. What is the upward-directed lift force exerted by the air on the airplane?

17. What is the net force acting on a 3900-lb automobile accelerating at 13 ft/s^2?

18. A 523-kg experimental rocket sled can be accelerated from rest to 1620 km/h in 1.82 s. What net force is required?

19. A jet plane starts from rest on the runway and accelerates for takeoff at 2.30 m/s^2 (= 7.55 ft/s^2). It has two jet engines, each of which exerts a thrust of 1.40×10^5 N (= 15.7 tons). What is the weight of the plane?

3-8 Applications of Newton's Laws

20. (*a*) Two 10-lb weights are attached to a spring scale as shown in Fig. 3-25*a*. What is the reading of the scale? (*b*) A single 10-lb weight is attached to a spring scale which itself is attached to a wall, as shown in Fig. 3-25*b*. What is the reading of the scale? (Ignore the weight of the scale.)

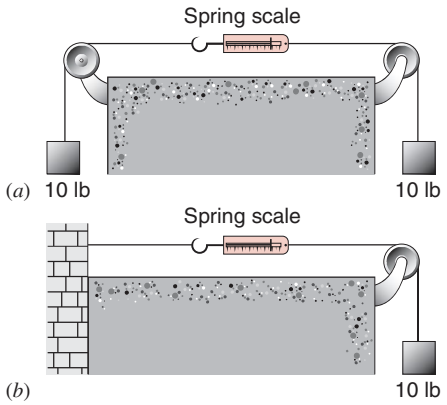

FIGURE 3-25. Exercise 20.

21. A car moving initially at a speed of 50 mi/h (≈80 km/h) and weighing 3000 lb (≈13,000 N) is brought to a stop in a distance of 200 ft (≈61 m). Find (*a*) the braking force and (*b*) the time required to stop. Assuming the same braking force, find (*c*) the distance and (*d*) the time required to stop if the car were going 25 mi/h (≈40 km/h) initially.

22. A meteor of mass 0.25 kg is falling vertically through Earth's atmosphere with an acceleration of 9.2 m/s^2. In addition to gravity, a vertical retarding force (due to the frictional drag of the atmosphere) acts on the meteor. What is the magnitude of this retarding force? See Fig. 3-26.

FIGURE 3-26. Exercise 22.

23. A man of mass 83 kg (weight 180 lb) jumps down to a concrete patio from a window ledge only 0.48 m (= 1.6 ft) above the ground. He neglects to bend his knees on landing, so that his motion is arrested in a distance of about 2.2 cm (= 0.87 in.). (*a*) What is the average acceleration of the man from the time his feet first touch the patio to the time he is brought fully to rest? (*b*) With what average force does this jump jar his bone structure?

24. What strength fishing line is needed to stop a 19-lb salmon swimming at 9.2 ft/s in a distance of 4.5 in.?

25. How could a 100-lb object be lowered from a roof using a cord with a breaking strength of 87 lb without breaking the cord?

26. An object is hung from a spring scale attached to the ceiling of an elevator. The scale reads 65 N when the elevator is standing still. (*a*) What is the reading when the elevator is moving upward with a constant speed of 7.6 m/s? (*b*) What is the reading of the scale when the elevator is moving upward with a speed of 7.6 m/s and decelerating at 2.4 m/s^2?

27. Workers are loading equipment into a freight elevator at the top floor of a building. However, they overload the elevator and the worn cable snaps. The mass of the loaded elevator at the time of the accident is 1600 kg. As the elevator falls, the guide rails exert a constant retarding force of 3700 N on the elevator. At what speed does the elevator hit the bottom of the shaft 72 m below?

28. A 26-ton Navy jet (Fig. 3-27) requires an air speed of 280 ft/s for lift-off. Its own engine develops a thrust of 24,000 lb. The jet is to take off from an aircraft carrier with a 300-ft flight deck. What force must be exerted by the catapult of the carrier? Assume that the catapult and the jet's engine each exert a constant force over the 300-ft takeoff distance.

FIGURE 3-27. Exercise 28.

29. A rocket and its payload have a total mass of 51,000 kg. How large is the thrust of the rocket engine when (*a*) the rocket is "hovering" over the launch pad, just after ignition, and (*b*) when the rocket is accelerating upward at 18 m/s^2?

30. A 77-kg person is parachuting and experiencing a downward acceleration of 2.5 m/s^2 shortly after opening the parachute. The mass of the parachute is 5.2 kg. (*a*) Find the upward force exerted on the parachute by the air. (*b*) Calculate the downward force exerted by the person on the parachute.

31. A 15,000-kg helicopter is lifting a 4500-kg car with an upward acceleration of 1.4 m/s^2. Calculate (*a*) the vertical force the air exerts on the helicopter blades and (*b*) the tension in the upper supporting cable; see Fig. 3-28.

FIGURE 3-28. Exercise 31.

PROBLEMS

1. A light beam from a satellite-carried laser strikes an object ejected from an accidentally launched ballistic missile; see Fig. 3-29. The beam exerts a force of 2.7×10^{-5} N on the target. If the "dwell time" of the beam on the target is 2.4 s, by how much is the object displaced if it is (*a*) a 280-kg warhead and (*b*) a 2.1-kg decoy? (These displacements can be measured by observing the reflected beam.)

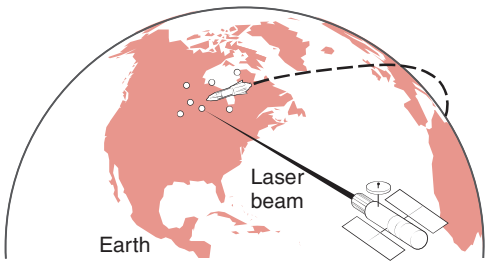

FIGURE 3-29. Problem 1.

2. A 40-kg girl and an 8.4-kg sled are on the surface of a frozen lake, 15 m apart. By means of a rope the girl exerts a 5.2-N force on the sled, pulling it toward her. (*a*) What is the acceleration of the sled? (*b*) What is the acceleration of the girl? (*c*) How far from the girl's initial position do they meet, presuming the force to remain constant? Assume that no frictional forces act.

3. A block is released from rest at the top of a frictionless inclined plane 16 m long. It reaches the bottom 4.2 s later. A second block is projected up the plane from the bottom at the instant the first block is released in such a way that it returns to the bottom simultaneously with the first block. (*a*) Find the acceleration of each block on the incline. (*b*) What is the ini-

tial velocity of the second block? (*c*) How far up the incline does it travel? You can assume that both blocks experience the same acceleration.

4. A 1400-kg jet engine is fastened to the fuselage of a passenger jet by just three bolts (this is the usual practice). Assume that each bolt supports one-third of the load. (*a*) Calculate the force on each bolt as the plane waits in line for clearance to take off. (*b*) During flight, the plane encounters turbulence, which suddenly imparts an upward vertical acceleration of 2.60 m/s^2 to the plane. Calculate the force on each bolt now. Why are only three bolts used? See Fig. 3-30.

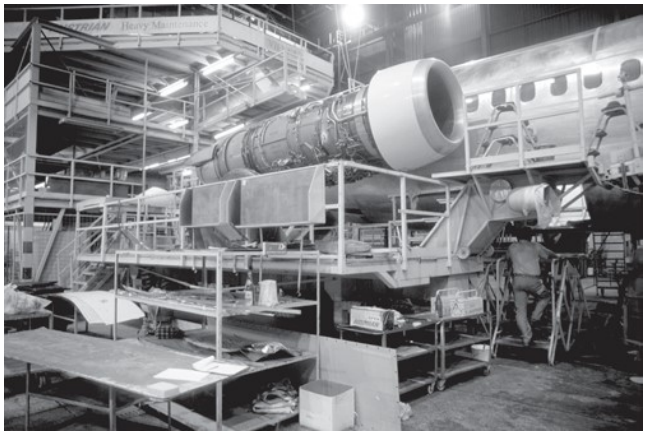

FIGURE 3-30. Problem 4.

5. A landing craft approaches the surface of Callisto, one of the satellites (moons) of the planet Jupiter (Fig. 3-31). If an upward thrust of 3260 N is supplied by the rocket engine, the craft descends with constant speed. Callisto has no atmosphere. If the upward thrust is 2200 N, the craft accelerates downward at 0.390 m/s^2. (*a*) What is the weight of the landing craft in the vicinity of Callisto's surface? (*b*) What is the mass of the craft? (*c*) What is the acceleration due to gravity near the surface of Callisto?

FIGURE 3-31. Problem 5.

6. A research balloon of total mass M is descending vertically with downward acceleration a (see Fig. 3-32). How much ballast must be thrown from the car to give the balloon an upward acceleration a, assuming that the upward lift of the air on the balloon does not change?

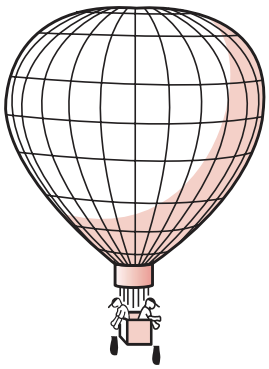

FIGURE 3-32. Problem 6.

7. A child's toy consists of three cars that are pulled in tandem on small frictionless rollers (Fig. 3-33). The cars have masses $m_1 = 3.1$ kg, $m_2 = 2.4$ kg, and $m_3 = 1.2$ kg. If they are pulled to the right with a horizontal force $P = 6.5$ N, find (*a*) the acceleration of the system, (*b*) the force exerted by the second car on the third car, and (*c*) the force exerted by the first car on the second car.

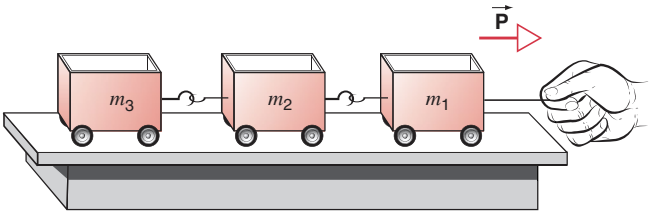

FIGURE 3-33. Problem 7.

8. Figure 3-34 shows three crates with masses $m_1 = 45.2$ kg, $m_2 = 22.8$ kg, and $m_3 = 34.3$ kg on a horizontal frictionless surface. (*a*) What horizontal force F is needed to push the crates to the right, as one unit, with an acceleration of 1.32 m/s^2? (*b*) Find the force exerted by m_2 on m_3. (*c*) By m_1 on m_2.

9. A chain consisting of five links, each with mass 100 g, is lifted vertically with a constant acceleration of 2.50 m/s^2, as shown in Fig. 3-35. Find (*a*) the forces acting between adja-

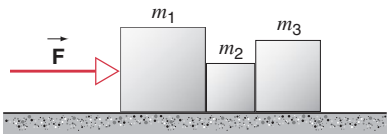

FIGURE 3-34. Problem 8.

cent links, (*b*) the force F exerted on the top link by the agent lifting the chain, and (*c*) the net force on each link.

FIGURE 3-35. Problem 9.

10. Two blocks are in contact on a frictionless table. A horizontal force is applied to one block, as shown in Fig. 3-36. (*a*) If $m_1 = 2.3$ kg, $m_2 = 1.2$ kg, and $F = 3.2$ N, find the force of contact between the two blocks. (*b*) Show that if the same force F is applied to m_2 rather than to m_1, the force of contact between the blocks is 2.1 N, which is not the same value derived in (a). Explain.

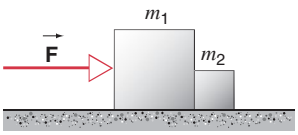

FIGURE 3-36. Problem 10.

11. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m, as shown in Fig. 3-37. A horizontal force $\vec{\mathbf{P}}$ is applied to one end of the rope. Assuming that the sag in the rope is negligible, find (*a*) the accelerationof rope and block, and (*b*) the force that the rope exerts on the block.

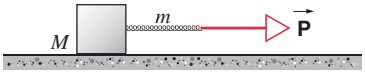

FIGURE 3-37. Problem 11.

COMPUTER PROBLEM

1. A 10.0 kg object is launched vertically into the air with an initial velocity of 50.0 m/s. In addition to the force of gravity there is a frictional force which is proportional to velocity according to $f_y = -bv_y$; note that this frictional force is negative (down) when the object is moving up, but positive (up) when the object is moving down.

(*a*) Numerically generate distance-time graphs for the object, using $b = 0$, but use several different step sizes for Δt, such as 1.0 s, 0.1 s, 0.01 s, and 0.001 s. Show the results on a single graph. How does the highest point vary with the step size?

(*b*) Numerically generate distance-time graphs for the object, using a step size of $\Delta t = 0.01$ s. Now, however, try non-zero values for b, such as 0.1 N·s/m, 0.5 N·s/m, 1.0 N·s/m, 5.0 N·s/m, and 10.0 N·s/m. How does the highest point vary with b? What do you notice about the shape of the graphs as b increases?